Capacity of All Nine Models of Channel Output Feedback for the Two-user Interference Channel

Achaleshwar Sahai, Vaneet Aggarwal, Melda Yuksel and Ashutosh Sabharwal

Abstract

In this paper, we study the impact of different channel output feedback architectures on the capacity of the two-user interference channel. For a two-user interference channel, a feedback link can exist between receivers and transmitters in 9 canonical architectures (see Fig. 2), ranging from only one feedback link to four feedback links. We derive the exact capacity region for the symmetric deterministic interference channel and the constant-gap capacity region for the symmetric Gaussian interference channel for all of the 9 architectures. We show that for a linear deterministic symmetric interference channel, in the weak interference regime, all models of feedback, except the one, which has only one of the receivers feeding back to its own transmitter, have the identical capacity region. When only one of the receivers feeds back to its own transmitter, the capacity region is a strict subset of the capacity region of the rest of the feedback models in the weak interference regime. However, the sum-capacity of *all* feedback models is identical in the weak interference regime. Moreover, in the strong interference regime all models of feedback with at least one of the receivers feeding back to its own transmitter have the identical sum-capacity. For the Gaussian interference channel, the results of the linear deterministic model follow, where capacity is replaced with approximate capacity.

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I. Introduction

The two-user interference channel has been studied in the literature since 1970's to understand one of the main performance limits of multiuser communication networks [3–9]. Feedback in interference channels has been considered in order to achieve a possible improvement in data rates. A large body of work on interference channels [10–13] explores feedback strategies, where each receiver sends channel output feedback to its own transmitter. More generalized form of feedback in a two-user interference channel is considered in [14–17]. Recent work in [18, 19] particularly analyzes the capacity region of two-user deterministic and Gaussian interference channels, where each of the receivers send channel output feedback to its own transmitter. The authors of [20] consider the case of rate limited channel output feedback and investigate its capacity region, where each user feeds back to its own transmitter.

The conventional model of channel output feedback in a two-user interference channel has each receiver feeding back to its intended transmitter [18–20]. However, several different feedback architectures are possible based on the presence or absence of feedback links between both receivers and both transmitters. The feedback architecture can be asymmetric if feedback resources available to different transmitter-receiver pairs are different. Consider two mobile terminals in two neighboring cells, communicating with their corresponding base stations. If the mobile user in the first cell is closer to its base-station, then its base-station can support a strong feedback link. At the same time, if the mobile station in the neighboring cell is farther away from its base-station, it will experience a poor or possibly no feedback channel. In such a case, we say only one direct-link feedback is available. In another scenario, suppose one of the receivers in the interference channel is capable of sending feedback to both the transmitters, whereas the other receiver does not send any feedback. Then it would be a case of single receiver broadcasting feedback. The conventional model of channel output feedback is *insufficient* to understand the effect of feedback on the capacity region of the interference channel. We need to consider different feedback architectures, which forms the focus of our study.

In this paper, we conduct a comprehensive study of the capacity region of all feedback architectures in two-user linear deterministic [21, 22] and Gaussian interference channels. The feedback architectures that we study are all

A. Sahai and A. Sabharwal are with the department of ECE, Rice University, Houston, TX 77005, USA (email: {as27,ashu}@rice.edu). V. Aggarwal is with AT&T Labs-Research, Florham Park, NJ 07932, USA (email: vaneet@research.att.com). M. Yuksel is with TOBB University of Economics and Technology, Ankara, Turkey (email: yuksel@etu.edu.tr). The material in this paper was presented in part at the IEEE Information Theory Workshop, Taormina, Italy, 2009 [1] and at the IEEE International Symposium on Information Theory, Austin, Texas, 2010 [2]. A. Sahai and A. Sabharwal were partially supported by NSF grant CNS-1012921 and a grant from Texas Instruments.

parametrized by the feedback links they support. In a two-user interference channel, there can be as many as 4 possible feedback links, i.e., one feedback link from each receiver to each transmitter. Therefore, excluding the case of no feedback links, a total of $2^4 - 1 = 15$ feedback models are possible. Barring the symmetrical cases, 9 canonical feedback models are possible, which are shown in Fig. 2. In this work, we study the capacity region of all the 9 feedback models shown in Fig. 2. In order to gain insights about good communication schemes that apply to the different feedback models, we first analyze them under the symmetric linear deterministic model of interference. Then, we extend the results to the Gaussian interference channel, deriving the approximate capacity region by developing outer and inner bounds, which are within constant bits of one another.

In this paper, the comprehensive study of capacity region of different feedback architectures leads to three main results. The first main result of the paper is that for a linear symmetric deterministic interference channel, all 9 canonical feedback models except one (with only one direct-link feedback, shown in Fig. 2(d)) have the identical capacity region in the weak interference regime. Moreover, the capacity region of single direct link feedback model is a strict subset of the capacity region of the rest of the feedback models. The first main result extends to the Gaussian channel case where all models of feedback, except single direct-link feedback model, have the same approximate capacity region which is within constant bits from their respective outer-bounds.

The second main result of the paper is that for a linear symmetric deterministic interference channel, all feedback models have the *identical* sum-capacity in the weak interference regime. This result is particularly interesting because if sum-capacity is the performance metric, any one feedback link is sufficient to achieve the maximum feedback sum-capacity.

The third main result of the paper is that to achieve maximum feedback sum-capacity, availability of one direct feedback link is sufficient for all regimes of interference, i.e., the sum-capacity with single direct feedback link is identical to the sum-capacity with all four feedback links for all regimes of interference. The second and third main results also hold for the Gaussian interference channel, if the term sum-capacity is replaced with approximate sum-capacity.

We show the above three results by deriving exact (deterministic)/approximate (Gaussian) capacity regions of all of the 9 canonical feedback models. We find two new outer-bounds and propose two new achievability schemes. For the deterministic channel model, the achievability scheme attains all points on the outer bound, whereas in the Gaussian model, the inner bound is a constant number of bits away from the outer bound (2.59 bits/Hz for feedback models in Fig. 2(a), 2(b), and 2(c), 4.59 bits/Hz for Fig. 2(d) and Fig. 2(e)). The achievability for all the feedback models is derived in two steps. First, an achievable strategy is proposed for two atomic feedback models: one with single direct feedback link and another with single cross feedback link (where one of the receivers feeds back to its interfering transmitter). Then, using a combination of the achievable strategies for the two atomic feedback models, the achievable rate region of the rest of the feedback models is derived.

The first achievable strategy we propose for single direct feedback link is based on using a Han-Kobayashi type message splitting [23]. Our coding strategy is similar to the one employed in two-user interference channel without feedback in the sense that the coding scheme splits the message at each transmitter into two parts, private and common. However, the coding strategy differs in the transmission of the common message. The common message generated at the second transmitter is transmitted twice, once by the transmitter, where it is generated, and once from the other transmitter, where it is known via feedback. The purpose of the re-transmission of the common message depends on the regime of interference. In the strong interference regime, feedback offers gain, if it allows the common message to travel from its source to destination via an alternate independent path of higher capacity (than the direct link). In the weak interference regime, the first transmitter can perform block-Markov encoding based on the common message of the second transmitter. Block-Markov encoding of messages based on the common message of the second transmitter, helps the first receiver to resolve some of the past interference, without causing any apparent interference at the second receiver.

The above achievable strategy turns out to be insufficient to show the exact/approximate capacity region for deterministic/Gaussian interference channel with feedback models shown in Fig. 2(e). The second achievable strategy, for single cross-link feedback model, is based on block-Markov encoding of messages at the second transmitter and dirty paper encoding at the first transmitter. Since the second transmitter performs block-Markov encoding, and cross-link feedback is available to the first transmitter, the first transmitter can learn about the "future" interference that its receiver will face. Based on the channel output feedback from the second receiver, the first transmitter performs dirty paper encoding to protect its receiver from future interference. Using this second achievable strategy in combination with the first achievable strategy, the capacity region for cross-link feedback is

proven.

Relations to similar work: The coding strategy in [16, 18, 24] also employ a Han-Kobayashi type message splitting. In [18], the feedback model has each transmitter receiving feedback from its respective receiver, and while the message is split into only two parts, private and common, only a part of the common message of the other transmitter is re-transmitted in subsequent blocks. Our coding scheme for the single direct-link feedback re-transmits all the common message of only one of the transmitters. In [16, 24], the message is split into four parts: two common and two private and feedback induces source cooperation by making sources learn the common message of the other transmitter. In our coding scheme too, the purpose of re-transmitting the common message is to induce cooperation/allow routing.

We would also like to remark that the work on generalized feedback in [14–17], as well as the work on source cooperation by two sources overhearing each other's messages over a noisy channel in [24] are closely related to our work. The outer and inner bounds derived in [14, 16] and [24], concurrent to our work in [1, 2], can be particularized to obtain the sum-capacity result shown in Lemma 4.4. In this work, we comprehensively study the exact and approximate capacity regions for linear deterministic and Gaussian interference channel models respectively for all canonical feedback models.

The rest of the paper is organized as follows. Section II introduces the Gaussian channel model and its deterministic approximation. Section II also presents all the different feedback models that will be studied in the paper. Section III is a preview of the main results and insights regarding them. Section IV and V present the capacity regions (exact and approximate respectively) for the linear deterministic and Gaussian interference channels for all models of feedback. Section VI concludes the paper with discussions.

II. Channel Model and Preliminaries

In this section, we describe the two-user symmetric Gaussian and deterministic interference channel models and the 9 canonical feedback architectures that will be used throughout the paper.

A. Channel Model

A two-user interference channel consists of two transmitters, T_1 and T_2, and two receivers D_1 and D_2. Each receiver D_u is interested in the message transmitted by transmitter T_u for $u \in \{1, 2\}$, while the message from the other transmitter is interference.

The two-user symmetric Gaussian interference channel, shown in Fig. 1(b) is a special case of the two-user interference channel, where the noise at both the receivers have zero mean, unit variance complex Gaussian distribution. Let W_u denote the message T_u transmits in N successive transmissions, where $W_u \in \mathcal{W}_u = \{1, 2 \dots 2^{NR_u}\}$, $N \in \mathbb{N}$ and $R_u \in \mathbb{R}$. The function $f_{uj} : W_u \mapsto X_{uj}$ denotes the encoding that maps the message to the input over the channel, $X_{uj} \in \mathbb{C}$, $j \in [1, 2, \dots N]$. Let $X_u^N = [X_{u1}, X_{u2}, \dots X_{uN}]$ and $Y_u^N = [Y_{u1}, Y_{u2}, \dots Y_{uN}]$, where X_{uj} (Y_{uj}) denotes the signal transmitted (received) at the j^{th} time instant at T_u (D_u). Then, when $g_{ij} \in \mathbb{C}$ are the channel gains, the received signals at the two receivers are given by

$$\begin{aligned} Y_{1j} &= g_{11}X_{1j} + g_{21}X_{2j} + Z_{1j} \\ Y_{2j} &= g_{22}X_{2j} + g_{12}X_{1j} + Z_{2j}. \end{aligned}$$

The decoding function h_u maps the output Y_u^N to a symbol $\widehat{W}_u \in \mathcal{W}_u$ ($h_u : Y_u^N \mapsto \widehat{W}_u$).

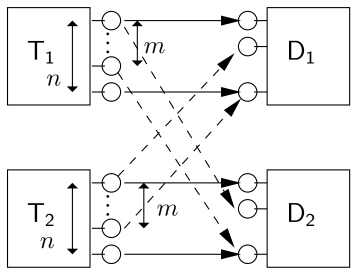
In this paper, we will focus on the symmetric Gaussian channel, where the direct gains are equal, $g_{11} = g_{22} = g_d$, the cross gains are equal, $g_{12} = g_{21} = g_c$, and the noises Z_{1j} and Z_{2j} are both distributed as $\mathcal{CN}(0, 1)$. Moreover, the transmitted power is constrained such that $\mathbb{E}(|X_{1j}|^2) \leq P_1$, $\mathbb{E}(|X_{2j}|^2) \leq P_2$, and $P_1 = P_2 = P$, where the $\mathbb{E}(.)$ denotes the expected value of a random variable. We also define the signal to noise ratio (SNR) and the interference to noise ratio (INR) as

$$\mathsf{SNR} = |g_d|^2 P, \ \ \mathsf{INR} = |g_c|^2 P.$$

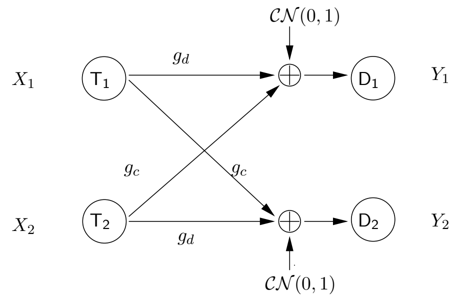
The regime of interference is weak, when $\mathsf{SNR} \geq \mathsf{INR}$ and strong when $\mathsf{SNR} < \mathsf{INR}$. Moreover, the ratio of INR to SNR in dB scale will be denoted by

$$\alpha = \frac{\log(\mathsf{INR})}{\log(\mathsf{SNR})}. \tag{1}$$

The deterministic interference channel [21] is a good approximation of the Gaussian interference channel, when signal and interference powers are much larger compared to the noise. We will use the deterministic approximation



(a)



(b)

Fig. 1. The (a) deterministic and (b) Gaussian models for the two-user interference channel.

of the two-user Gaussian interference channel with feedback to develop insights for designing achievable communication strategies for the Gaussian model. The deterministic interference channel is described as follows. Associated with the link between transmitter T_u, $u \in \{1,2\}$, and receiver D_k, $k \in \{1,2\}$, is a non-negative integer n_{uk} (which corresponds to the channel gain in the Gaussian channel). Let $q = \max_{u,k}(n_{uk})$. Overloading the notation for input and output, the inputs at u^{th} transmitter at time j is denoted by $X_{uj} \in \mathbb{F}_2^q$. Equivalently, X_{uj} can be written as $X_{uj} = \left[X_{uj_1} X_{uj_2} \ldots X_{uj_q}\right]^T$, such that X_{uj_1} and X_{uj_q} are the most and the least significant bits respectively. The received signal at time j is denoted by the vector $Y_{kj} \in \mathbb{F}_2^q$ or equivalently $Y_{kj} = \left[Y_{kj_1} Y_{kj_2} \ldots Y_{kj_q}\right]^T$. Specifically, the received signal Y_{kj}, $k = 1, 2$, of a deterministic interference channel is given by

$$Y_{kj} = \mathbf{S}^{q-n_{1k}} X_{1j} \oplus \mathbf{S}^{q-n_{2k}} X_{2j} \quad k = \{1,2\}, \tag{2}$$

where $\oplus$ denotes the XOR operation, and $\mathbf{S}$ is a $q \times q$ shift matrix with ones on the first diagonal below the main diagonal, and zeros everywhere else. The symmetric deterministic channel, shown in Fig. 1(a), is characterized by two values: $n = n_{11} = n_{22}$ and $m = n_{12} = n_{21}$. Here n and m indicate the number of signal bit levels that we can send through the direct links and the cross links, respectively. When $\frac{m}{n} \leq 1$, the system is in the weak interference regime, and when $\frac{m}{n} > 1$, the system is in the strong interference regime. We denote by $\mathbf{O}_p = [0, 0, \ldots, 0]^T$ such that the cardinality of $\mathbf{O}_p$ is p.

B. Feedback Models

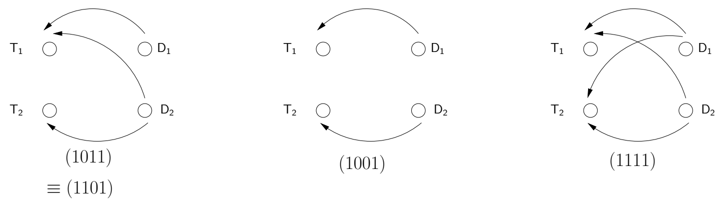
In this paper, we will use feedback to imply channel output feedback from the receivers to the transmitters. The feedback is assumed to be strictly causal and noiseless. There are four feedback links from the two receivers to the two transmitters. A feedback model is defined by the four-tuple $(F_{11} F_{12} F_{21} F_{22})$, where

$$F_{ku} = \begin{cases} 1 & \text{if there is a feedback link from } \mathsf{D}_k \text{ to } \mathsf{T}_u, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

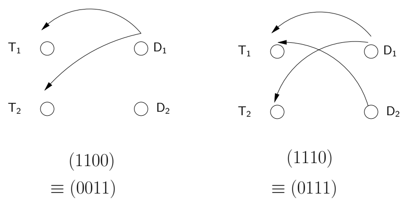
Fig. 2 shows the 9 principal feedback combinations and lists their symmetrical equivalent feedback models. With feedback, we can formalize the transmitted symbols as

$$X_{uj} = f_{uj}(W_u, Y_1^{j-1} F_{1u}, Y_2^{j-1} F_{2u}),\ u = \{1,2\},$$

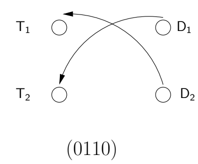
where $F_{ku} = 1$ implies that the channel output, Y_k^{j-1}, is known causally to the u^{th} transmitter. The feedback link from a receiver to its own transmitter is the direct-link feedback, and the link to the other transmitter is the cross-link feedback. If only two direct-links of feedback exist, then $F_{11} = F_{22} = 1$ and $F_{12} = F_{21} = 0$. If only one direct-link feedback exists, then $F_{12} = F_{21} = 0$ and either F_{11} or F_{22} is 1 while the other is zero. Since we consider the symmetric interference channel, unless otherwise specified, we will, without loss of generality, assume that one direct-link feedback model is equivalent to $F_{11} = 1$ and $F_{22} = 0$. When feedback is broadcast from a single receiver, we will assume that $F_{11} = F_{12} = 1$ while $F_{21} = F_{22} = 0$. In the (1111) feedback model, both receivers broadcast their channel outputs.



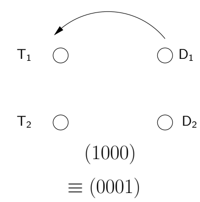
(a) Feedback models with at least two direct feedback links



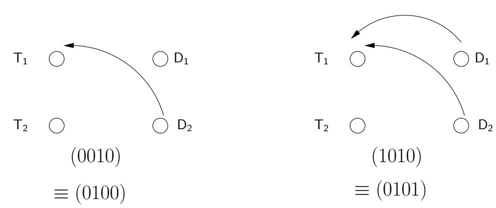
(b) One of the receivers is broadcasting feedback



(c) Both receivers feeding back to the unintended transmitter



(d) Only one receiver feeding back to its transmitter



(e) Feedback models where only one of the transmitter receives cross-link or cross- as well as direct-link feedback

Fig. 2. The 9 canonical feedback models. The figure shows only the feedback links, while the underlying interference channel is depicted in Fig. 1. The feedback state of each of the feedback models is also shown. These 9 models (15 including the symmetric cases) constitute all possible cases of feedback.

C. Achievable Rate and Capacity Definitions

A rate pair (R_1, R_2) is said to be *achievable*, if for independent and identically distributed (i.i.d.) messages $W_1 \in \mathcal{W}_1$ and $W_2 \in \mathcal{W}_2$, where $\mathcal{W}_u = \{1, \cdots, 2^{NR_u}\}$ and $u \in \{1, 2\}$, there exist encoders f_{uj} and decoders h_u so that the probability that the decoded messages $\widehat{W}_1$ and $\widehat{W}_2$ at D_1 and D_2 respectively are in error goes to 0 as $N \to \infty$. More precisely, for $u = \{1, 2\}$ define the average error probability of the message T_u transmits to D_u as

$$\epsilon_{u,N} = \mathbb{E}(\Pr(\widehat{W}_u \neq W_u)). \tag{4}$$

Then the rate pair (R_1, R_2) is said to be achievable, if both $\epsilon_{1,N}$ and $\epsilon_{2,N}$ can be driven to zero as $N \to \infty$. The capacity region is the closure of all achievable rate pairs (R_1, R_2). Since there are different capacity regions for different feedback models, we will use a superscript representing the state $(F_{11}F_{12}F_{21}F_{22})$. The capacity region and the sum-capacity of the $(F_{11}F_{12}F_{21}F_{22})$ feedback model are respectively denoted by $\mathcal{C}^{(F_{11}F_{12}F_{21}F_{22})}$ and $C_{\text{sum}}^{(F_{11}F_{12}F_{21}F_{22})}$, while the achievable rate region and the sum-rate are denoted by $\mathcal{R}^{(F_{11}F_{12}F_{21}F_{22})}$ and $R_{\text{sum}}^{(F_{11}F_{12}F_{21}F_{22})}$ respectively.

D. Prior Results

To contrast the capacity region and the sum-capacity results derived in this paper to the no feedback case, the following theorem is presented:

Theorem 2.1 ([6, 9, 22]): The capacity region of the two-user symmetric deterministic interference channel without any feedback, $\mathcal{C}^{(0000)}$ is the closure of all (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq n && (5)\\ R_2 &\leq n && (6)\\ R_1 + R_2 &\leq \min((n-m)^+ + \max(m,n), 2\max(m,(n-m))) && (7)\\ R_1 + 2R_2 &\leq \max(m,n) + (n-m)^+ + \max(m,(n-m)) && (8)\\ 2R_1 + R_2 &\leq \max(m,n) + (n-m)^+ + \max(m,(n-m)). && (9) \end{aligned}$$

The capacity region of the two-user deterministic interference channel, with feedback from both the receivers to their respective transmitters, i.e. the (1001) feedback model, has been studied in [18, 19] and is given by:

Theorem 2.2 ([19]): The capacity region of the two-user symmetric deterministic interference channel with two direct feedback links, $\mathcal{C}^{(1001)}$, is the closure of all (R_1, R_2) satisfying

$$\begin{aligned} R_1 &\leq \max(n,m) && (10)\\ R_2 &\leq \max(n,m) && (11)\\ R_1 + R_2 &\leq (n-m)^+ + \max(n,m). && (12) \end{aligned}$$

Theorem 2.2 shows that with feedback, the capacity region of the deterministic interference channel enlarges and the sum-capacity improves, as the (1001) feedback model deactivates the bounds (8) and (9) in Theorem 2.1.

III. Preview of Main Results

In this paper, we will prove the capacity/approximate-capacity region of all 9 canonical feedback models for the deterministic/Gaussian channels. Before presenting the technical details in Sections IV and V, in this section, we highlight our main results and offer related insights.

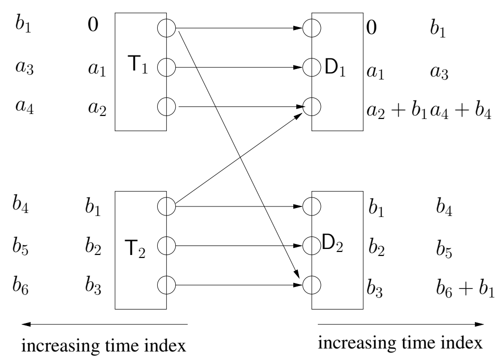
1) Except the single direct-link feedback model (1000), all feedback models have the identical capacity region in the weak interference regime. Thus, all feedback models (except the (1000) feedback model) achieve the capacity region achievable by all four feedback links, $\mathcal{C}^{(1111)}$. In particular, this result includes that the capacity region of the single cross-link feedback model is identical with the capacity region with all four feedback links, i.e., $\mathcal{C}^{(0010)} \equiv \mathcal{C}^{(1111)}$ in the weak interference regime. Moreover, the capacity region $\mathcal{C}^{(1000)}$ is a strict subset of $\mathcal{C}^{(1111)}$.
2) All feedback models have the identical sum-capacity in the weak interference regime.
3) In the strong interference regime, feedback models with at least one direct-link of feedback have the same sum-capacity as that with all four feedback links, i.e., $C_{\text{sum}}^{(1000)} = C_{\text{sum}}^{(1\text{xxx})} = C_{\text{sum}}^{(1111)}$.

The above results that hold for deterministic channels apply to Gaussian channels, if the term capacity is replaced with approximate capacity. We develop the above results by deriving two new outer-bounds and proposing two new achievability schemes. An illustration of the achievability schemes through examples and intuitions about the above results follow.

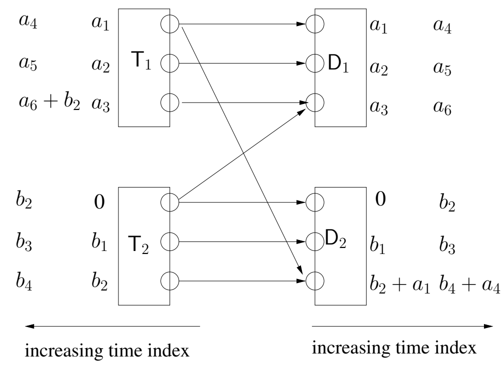
A. Weak Interference Regime

Gain due to source cooperation: If a source receives feedback, it can *causally* learn a part of the message being transmitted by the other source. Thus source cooperation can be induced, which improves the capacity region and the sum-capacity. For instance, let T_1 receive feedback, then it can causally learn a part of the message transmitted by the interfering source, T_2. If the "past" interference impairs decoding the intended signal at D_1, then with the help of causal feedback, T_1 can encode its message in the forthcoming blocks such that it can help its receiver resolve the "past" interference. On the other hand, the knowledge of the message transmitted by T_2 can also be used by T_1 to encode its message such that it is robust against "future" interference from T_2. We illustrate the two forms of source cooperation, which are possible in direct-link and cross-link feedback models through two examples in a deterministic model with $\frac{m}{n} = \frac{1}{3}$.

Example 1, direct-link feedback: In the coding strategy shown in Fig. 3(a), T_1 learns the interference, b_1, received at D_1 in the first block via feedback and transmits it in the second block on a bit-level that is above its interference floor. This enables D_1 to decode the interference that occurred in the first block, while causing no



(a) Using the direct-link (1000) feedback model, b_1 received in the second block at D_1 helps resolve the interference at D_1 in the first block. Also note that all the intended bits can be decoded at D_2, and thus there is no interference observed at D_2. The rate pair $(2, 3)$ is achievable, when number of blocks $\to \infty$.



(b) Using the cross-link (0010) feedback model, b_2 is known at T_1 at the end of the first block of transmission. Using the knowledge of b_2, dirty paper encoding is performed at T_1, such that the interference from T_2 does not impair reception at D_1. The rate pair $(3, 2)$ is achievable, when number of blocks $\to \infty$.

Fig. 3. The first two blocks of encoding for (1000) and (0010) feedback models for the deterministic interference channel with $n = 3, m = 1$.

apparent interference at D_2 (since b_1 is an intended signal at D_2). With the number of blocks approaching ∞, the rate pair $(2, 3)$ is achievable.

Example 2, cross-link feedback: In the coding strategy shown in Fig. 3(b), T_1 learns the message transmitted by T_2 in the first block. The transmitter T_2 follows a block-Markov type encoding and repeats b_2 in the second time block. Since T_1 knows b_2 at the end of first block, via cross-link feedback, and is also aware that b_2 is the likely interference in the second block, it performs a dirty paper like encoding scheme to ensure that its message is robust to interference. In the second block three bits of intended message are decodable at D_1 and two bits are decodable at D_2, thus leading to rate pair $(3, 2)$ as number of blocks $\to \infty$.

In either of the examples above, one of the transmitter-receiver pairs communicates essentially "interference-free", even though the other transmitter is transmitting at bit-levels which cause interference. Such interference-free communication is impossible without feedback, unless one of the transmitter-receiver pair sacrifices its rate. An important difference between direct and cross-link feedback is that with direct-link feedback, T_1 can know only the "past" interference, while with cross-link feedback T_1 has access to possibly both "past" and "future" interference. Thus, with cross-link feedback, the rate pairs $(3, 2)$ as well as $(2, 3)$ are achievable if $\frac{m}{n} = \frac{1}{3}$. However, with single direct-link feedback, where T_1 is the only source receiving feedback, the rate pair $(3, 2)$ is *not* achievable. Therefore, in the weak interference regime, cross-link feedback model has a larger capacity region than direct-link link feedback, $\mathcal{C}^{(1000)} \subset \mathcal{C}^{(0010)}$. Also, if both direct feedback links are present, then by symmetry both rate pairs $(3, 2)$ as well as $(2, 3)$ are achievable. Thus, $\mathcal{C}^{(1000)} \subset \mathcal{C}^{(1001)}$.

Limited gain due to feedback delay: Feedback implies that cooperation between sources can occur only causally. In the example shown in Fig 3(a), T_1 expends resources (bits) to help its receiver resolve "past" interference, while in the example shown in Fig 3(b), T_2 expends resources in creating known interference at D_1. Even with all four feedback links present, i.e., (1111) feedback model, there is a trade-off between expending resources to transmit a new message versus resolving past interference/creating known interference. Therefore, in the weak interference regime, having all four feedback links does not enlarge the capacity region compared to two direct-link feedback or cross-link feedback, i.e., $\mathcal{C}^{(0010)} \equiv \mathcal{C}^{(1001)} \equiv \mathcal{C}^{(1111)}$.

Equivalence of sum-capacity: The capacity region of the single direct-link feedback model is smaller than the rest of the feedback models. This is so because in single direct-link feedback model, unlike other feedback models, only one of the sources, say T_1, can assist the other source, T_2 such that there is no apparent interference at its intended receiver D_2. However, such one-sided assistance is sufficient to achieve the same sum-capacity as would be achievable with two sided assistance (possible with cross-link, two direct-link or all four feedback links). Thus, $C_{\text{sum}}^{(1000)} = C_{\text{sum}}^{(1\text{xxx})} = C_{\text{sum}}^{(0010)} = C_{\text{sum}}^{(1111)}$.

(a) The dashed line indicates the alternate path from T_2 to D_2.

(b) Both alternate paths for communicating a message from a transmitter to its intended receiver are depicted, one is the dashed line and the other is the dotted line. The alternate paths share a common, finite capacity sub-path.

Fig. 4. Alternate paths, which improve rates in the strong interference regime.

B. Strong Interference Regime

In the strong interference regime, feedback offers improvement in both the sum rate and the capacity region, if it enables an alternate independent path of higher capacity for messages to travel from its source to its destination. As a direct consequence, any feedback model, which does not lead to an alternate path, attains no gain. On the other hand, in feedback models, which obtain gains out of feedback (models with at least one direct feedback link), in the strong interference regime, the gain is limited by the capacity of the alternate path. We describe how this limitation leads to the result that all feedback models with at least one direct feedback link have the same sum-capacity.

Gain due to alternate independent path: In Fig. 4(a), single direct-link feedback enables an alternate independent path for messages to travel from T_2 to D_2. The feedback link between D_1 and T_1, in conjunction with the interfering links between T_2-D_1 and T_1-D_2 forms the alternate path. Since the interfering links are stronger than the direct link and feedback is of infinite capacity, the rate at which T_2-D_2 can communicate is higher than the rate possible without feedback. Note that, only T_2-D_2 pair has an alternate independent path, and therefore only the rate of T_2-D_2 increases. On the other hand, in the example shown in Fig. 4(b), with two direct feedback links, both transmitter-receiver pairs have alternate independent paths. Consequently rates of both source-destination pairs can be boosted. Therefore, the capacity region achievable with the feedback model with both direct feedback links is larger than the capacity region achievable with the feedback model with only one direct feedback link, i.e., $\mathcal{C}^{(1000)} \subset \mathcal{C}^{(1001)}$.

A key commonality in the feedback models shown in Fig. 4(a) and Fig. 4(b) is that the alternate independent path in both feedback models necessarily contains the pair of interfering links, T_1-D_2 and T_2-D_1 as a resource that is intelligently used to boost the rate. The increase in the sum rate is limited by the capacity of the shared resource, i.e. the capacity of the interference links. Thus, whether there is single direct feedback link or two direct feedback links, the same gain in the sum-rate is possible, thus the sum-capacity of all feedback models with at least one direct feedback link are identical, i.e., $C_{\text{sum}}^{(1000)} = C_{\text{sum}}^{(1\times\times\times)} = C_{\text{sum}}^{(1111)}$.

Cross link feedback creates no alternate path: Any model of feedback, which has only cross feedback links, does not result in any alternate independent path for messages to travel from the source to its destination. Consequently, no improvement in the individual rate or in the rate region is observed. Thus, the capacity region with or without cross link feedback are the same in the strong interference regime, i.e. $\mathcal{C}^{(0110)} \equiv \mathcal{C}^{(0000)}$. As the cross links do not bring in any gains, in the strong interference regime, the capacity region of the feedback model with all four feedback links is the same as the capacity region of the feedback model with only two direct feedback links, i.e. $\mathcal{C}^{(1\times\times1)} \equiv \mathcal{C}^{(1111)}$.

IV. Feedback models: Deterministic channels

In this section, we first present Theorem 4.1 and Theorem 4.2, which respectively state the capacity region and the sum-capacity of all 9 canonical feedback models for the linear deterministic channel. In Section IV-A, we provide outer-bounds on the capacity region and the sum-capacity in Lemma 4.3 [24], Lemma 4.4 [1] and Lemma 4.5.

In Sections IV-B, IV-C and IV-D, we show the achievability of the capacity region and the sum-capacity of all 9 feedback models.

Theorem 4.1: The capacity regions of the two-user symmetric deterministic interference channel for all the 9 canonical feedback models are given in Table I.

TABLE I
CAPACITY REGIONS FOR ALL 9 CANONICAL FEEDBACK MODELS

Feedback Models	Capacity Region
(1xx1)	$R_1 \leq \max(n,m)$ $R_2 \leq \max(n,m)$ $R_1 + R_2 \leq (n-m)^+ + \max(n,m)$
(1100), (1110) (1010)	$R_1 \leq n$ $R_2 \leq \max(n,m)$ $R_1 + R_2 \leq (n-m)^+ + \max(n,m)$
(0110), (0010)	$R_1 \leq n$ $R_2 \leq n$ $R_1 + R_2 \leq (n-m)^+ + \max(n,m)$
(1000)	$R_1 \leq n$ $R_2 \leq \max(n,m)$ $R_1 + R_2 \leq (n-m)^+ + \max(n,m)$ $2R_1 + R_2 \leq (n-m)^+ + \max(n,m) + \max(n-m,m)$

Theorem 4.2: The sum-capacity of the two-user symmetric deterministic interference channel for all the 9 canonical feedback models is given in Table II.

TABLE II
DETERMINISTIC SUM-CAPACITY FOR ALL 9 CANONICAL FEEDBACK MODELS

Feedback Models	Sum-capacity
(1xxx)	$(n-m)^+ + \max(n,m)$
(0110), (0010)	$\min\{(n-m)^+ + \max(n,m), 2n\}$

A. Outer Bounds

Feedback in interference channels is a special case of source cooperation. Thus the cut-set bounds on the interference channel with source cooperation apply to interference channels with feedback as well. In this subsection, along with the cut-set bound for interference channels with feedback, we describe two new outer-bounds for different feedback models.

Lemma 4.3 ([24, 25]): The cut-set and no-interference bound for different feedback combinations is given by

$$R_1 \leq \max(n, c_1) \tag{13}$$
$$R_2 \leq \max(n, c_2), \tag{14}$$

where

$$c_1 = \begin{cases} 0 & \text{if } \mathsf{T}_2 \text{ receives no direct-link feedback} \\ m & \text{otherwise} \end{cases} \tag{15}$$

and

$$c_2 = \begin{cases} 0 & \text{if } \mathsf{T}_1 \text{ receives no direct-link feedback} \\ m & \text{otherwise} \end{cases}. \tag{16}$$

Next, we present an outer-bound on the sum-capacity of the feedback model (1111) we derived in [1]. Concurrent to [1], the authors in [15] derive an outer-bound for the generalized feedback model. The authors in [24] also derive outer bounds for interference channels with source cooperation, which can be particularized for the linear symmetric deterministic interference channel to obtain the same result.

Lemma 4.4 ([1, 15, 24]): The sum-capacity of the feedback model (1111), $\mathcal{C}_{\text{sum}}^{(1111)}$, is outer bounded by

$$R_1 + R_2 \leq (n-m)^+ + \max(n,m). \tag{17}$$

Remark 1: Since none of the feedback models can have a sum-capacity larger than the sum-capacity for the (1111) feedback model, (17) is an outer-bound on the sum-capacity of all feedback models.

Lemma 4.5: The capacity region of the two-user symmetric deterministic interference channel with feedback state (1000) is outer bounded by

$$2R_1 + R_2 \leq (n-m)^+ + \max(n,m) + \max(m, n-m). \tag{18}$$

Proof: The proof is provided in Appendix A. ■

Remark 2: We note that (18) is identical to (9), i.e., the bound on $2R_1 + R_2$, when there is no feedback.

B. *Achievability for Two Atomic Feedback Models*

To show the achievability of the capacity region of all 9 canonical cases of feedback, we first show an achievable strategy for the single direct-link feedback model, which is based on Han-Kobayashi type message splitting [23]. Then, to show the achievability of the single cross-link feedback model, we present Lemma 4.6, which allows us to connect the achievable rate region of the single direct-link feedback model with the achievable rate-region of the single cross-link feedback model. To complete the achievability of the single cross-link feedback model, we show another achievable strategy, which is based on block-Markov and dirty paper encoding and decoding. Finally, using the achievability for the single direct-link feedback and single cross-link feedback models, we show the achievability of the capacity region of all 9 canonical feedback models.

To show the achievability of the capacity region of single direct-link and single cross-link feedback models, we establish the achievability of the corner points formed by the intersection of the outer-bounds given by (13), (14), (17) and (18). Since the capacity regions are formed by the intersection of hyper-planes, the capacity regions are convex polygons. The achievability of a convex polygon is proved, if the non-trivial corner points of the convex polygon are shown to be achievable. We define the following points

$$\begin{aligned} \mathcal{K}_{\text{A}} &= \{(R_1, R_2)\text{: (13) and (18) hold with equality simultaneously}\}, \\ \mathcal{K}_{\text{B}} &= \{(R_1, R_2)\text{: (13) and (17) hold with equality simultaneously}\}, \\ \mathcal{K}_{\text{C}} &= \{(R_1, R_2)\text{: (17) and (18) hold with equality simultaneously}\}, \\ \mathcal{K}_{\text{D}} &= \{(R_1, R_2)\text{: (14) and (17) hold with equality simultaneously}\}, \\ \mathcal{K}_{\text{E}} &= \{(R_1, R_2)\text{: (13) and (14) hold with equality simultaneously}\}. \end{aligned} \tag{19}$$

(20)

1) Achievability for the (1000) *Feedback Model:* The outer-bounds for the (1000) feedback model are given by (13), (14), (17) and (18). It is easy to verify that in the weak interference regime, among the corner points, the corner points $\mathcal{K}_{\text{A}}$, $\mathcal{K}_{\text{C}}$ and $\mathcal{K}_{\text{D}}$ form the tightest outer bound, while in the strong interference regime, among the corner points, $\mathcal{K}_{\text{B}}$ and $\mathcal{K}_{\text{D}}$ describe the tightest outer bound. The achievability is as follows:

Encoding: The messages to be transmitted at both the transmitters are split into common and private parts. The common and private messages transmitted from the u^{th} transmitter, T_u, after encoding as channel inputs are denoted as $X_{ui,c}$ and $X_{ui,p}$. The corresponding rates are R_{uc} and R_{up}. The common message generated at T_2, $X_{2i-1,c}$, is learned by T_1 through feedback before the i^{th} block of transmission and re-transmitted by T_1 in the i^{th} block. By re-transmitting T_2's common message, T_1 performs a relaying action. The achievable rates are given by $R_1 = R_{1c} + R_{1p}$ and $R_2 = R_{2c} + R_{2p}$. The encoding runs for $B \to \infty$ blocks.

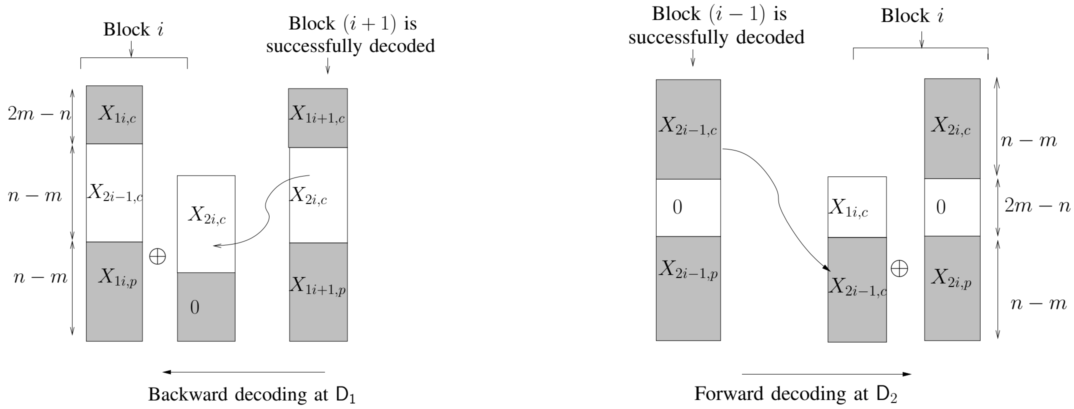


Fig. 5. Achievability of $(R_1, R_2) = (m, 2n-2m)$ with a single direct-link feedback. It lies on the boundary of the sum-rate upper bound (Lemma 4.4). At either of the receivers, signals learned after being decoded can be subtracted out to further decode the rest of the signals.

Decoding: To ensure reliable decoding, T_1 remains silent in the first block and T_2 remains silent in the last block. At D_1, backward decoding is applied, where the common message of T_2 is decoded starting B^{th} block. Thus, at D_1, before the i^{th} block is decoded $X_{2i,c}$ is known. Then $X_{2i,c}$ is subtracted from the received message Y_{1i}, assisting in decoding $X_{1i,c}$, $X_{2i-1,c}$ and $X_{1i,p}$. At D_2, since forward decoding is applied, decoding in the $(i-1)^{\text{th}}$ block yields $X_{2i-1,c}$ which is then subtracted from the received message Y_{2i} to decode $X_{1i,c}$, $X_{2i,c}$ and $X_{2i,p}$. In Fig. 5 two intermediate blocks of received messages at the two receivers are shown.

Rate allocation: In the weak interference regime, the corner point $\mathcal{K}_{\text{D}} \equiv (n-m, n)$ is not achievable without feedback [6, 9, 22]. Using the achievability described above, the rate pair $(n-m, n)$ is achievable. In the weak interference regime, where $\frac{m}{n} \leq 1$, both transmitters transmit $(n-m)$ bits of private message. Transmitter T_2 additionally transmits m bits of common message. As $B \to \infty$, the rates

$$R_1 = \underbrace{n-m}_{\text{private}},\; R_2 = \underbrace{n-m}_{\text{private}} + \underbrace{m}_{\text{common}} = n \tag{21}$$

are achievable. The corner point $\mathcal{K}_{\text{C}} \equiv (m, 2n-2m)$ is achievable without any feedback except when $\frac{1}{2} \leq \frac{m}{n} \leq \frac{2}{3}$. When $\frac{1}{2} \leq \frac{m}{n} \leq \frac{2}{3}$, in order to achieve the rate-pair $(m, 2n-2m)$, the private and common message rates are set as

$$R_1 = \underbrace{n-m}_{\text{private}} + \underbrace{2m-n}_{\text{common}} = m,\; R_2 = \underbrace{n-m}_{\text{private}} + \underbrace{n-m}_{\text{common}} = 2n-2m. \tag{22}$$

The corner point $\mathcal{K}_{\text{A}}$ is achievable without any feedback [6, 9, 22].

In the strong interference regime, the corner point $\mathcal{K}_{\text{D}} \equiv (0, m)$ is not achievable without feedback. However, with direct-link feedback, the T_1 - D_1 pair along with the feedback link, can be used as virtual relay node. More precisely, setting the rates

$$R_1 = 0,\; R_2 = \underbrace{m}_{\text{common}} \tag{23}$$

as $B \to \infty$, the rate pair $(0, m)$ bits per block can be achieved. The other non-trivial corner point $\mathcal{K}_{\text{B}} \equiv (n, m-n)$ is not achievable without feedback when $m > 2n$. Using feedback, when $m > 2n$, by setting rates

$$R_1 = \underbrace{n}_{\text{common}},\; R_2 = \underbrace{m-n}_{\text{common}}. \tag{24}$$

the desirable rate pair is achievable as $B \to \infty$. The detailed rate allocation strategy is described in Appendix B, which shows the achievability of the capacity region of the (1000) feedback model shown in Table I.

2) Relating $\mathcal{C}^{(1000)}$ and $\mathcal{C}^{(0010)}$: In order to re-use the achievability for the (1000) feedback model described above, in feedback models, which do not have a direct-link feedback, we show that the capacity regions satisfy $\mathcal{C}^{(1000)} \subseteq \mathcal{C}^{(0010)}$.

Lemma 4.6: The capacity region of the single cross-link feedback and single direct-link feedback, for $n \geq m$, are related as

$$\mathcal{C}^{(1000)} \subseteq \mathcal{C}^{(0010)} \tag{25}$$

$$\mathcal{C}^{(0001)} \subseteq \mathcal{C}^{(0100)}. \tag{26}$$

Proof: Due to the symmetry of the channel, it is sufficient to prove only one of the above inequalities. We prove (25). For the single direct-link feedback model, (1000), the encoding is constrained such that

$$X_{1i} = f_{1i}(W_1, Y_1^{i-1}),\ X_{2i} = f_{2i}(W_1). \tag{27}$$

In the cross link feedback model

$$X_{1i} = g_{1i}(W_1, Y_2^{i-1}), X_{2i} = g_{2i}(W_1). \tag{28}$$

Here, $Y_2^{i-1} = X_2^{i-1} \oplus V_1^{i-1}$, where $V_{1i} = \mathbf{S}^{q-m} X_{1i}$ is the interfering part of the transmitted message from T_1. Since X_1^{i-1} is known to T_1 before the i^{th} block of encoding, V_1^{i-1}, which is a subset of X_1^{i-1} is also known to T_1. With the cross-link feedback, since T_1 has access to Y_2^{i-1} before the i^{th} block of encoding, it can obtain X_2^{i-1}. Now, V_2^{i-1} is a subset of X_2^{i-1} (since $m \leq n$), and $Y_1^{i-1} = X_1^{i-1} \oplus V_2^{i-1}$. Thus, knowing Y_2^{i-1}, T_1 can form $Y_1^{i-1} = (X_1^{i-1} \oplus \mathbf{S}^{n-m}(Y_2^{i-1} \oplus \mathbf{S}^{n-m} X_1^{i-1}))$. Thus, for every message pair (W_1, W_2), and encoding function (f_{1i}, f_{2i}), choosing $g_{1i} \equiv f_{1i}$ and $g_{2i} \equiv f_{2i}$, the encoding operations defined in (27) and (28) can be made identical. Identical decoding naturally follows. Therefore,

$$\mathcal{C}^{(1000)} \subseteq \mathcal{C}^{(0010)}. \tag{29}$$

■

Remark 3: The result in Lemma 4.6 is based on the simple observation that when $n \geq m$ in the (0010) feedback model, the transmitter T_1 receives a "better" copy of the message encoded at T_2 than in the (1000) feedback model. This is because, in the (0010) feedback model, the feedback is received from D_2, while in the (1000) model, feedback is received from D_1. At D_2 and D_1, the received signals are linear combinations of X_{1i} and X_{2i}. At T_1, X_{1i} is known. As $n \geq m$, the bits of X_{2i} that can be decoded from the received message at D_2 form a superset of the bits of X_{2i} that can be decoded from the received message at D_1.

Corollary 4.7: When $n \geq m$, the capacity regions $\mathcal{C}^{(1001)}$, $\mathcal{C}^{(1100)}$, $\mathcal{C}^{(0110)}$ are related as follows

$$\mathcal{C}^{(1001)} \subseteq \mathcal{C}^{(1100)} \tag{30}$$

$$\mathcal{C}^{(1001)} \subseteq \mathcal{C}^{(0110)}. \tag{31}$$

Proof: In the (1001) feedback model, before the i^{th} block of encoding, T_1 and T_2 have access to Y_1^{i-1} and Y_2^{i-1} through feedback. In the (1100) feedback model, T_1 has access to Y_1^{i-1} and T_2 has also access to Y_1^{i-1} before the i^{th} block of encoding. As shown in Lemma 4.6, in the weak interference regime, using Y_1^{i-1}, T_2 can construct Y_2^{i-1}. Therefore, the achievable rate-region of the (1100) feedback model is at least as large as that of the (1001) feedback model. Thus, $\mathcal{C}^{(1001)} \subseteq \mathcal{C}^{(1100)}$. Similar proof follows for (31). ■

3) Achievability for the (0010) Feedback Model: The outer bound on the (0010) feedback model is characterized by the corner points $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$. From Theorem 4.1, we know that in the weak interference regime, the corner point $\mathcal{K}_{\mathrm{B}} \equiv (n, n-m)$ is outside the the boundary of $\mathcal{C}^{(1000)}$. Thus the achievability of the (1000) feedback model and Lemma 4.6 is not sufficient to show the achievability of the rate pair $(n, n-m)$ for the (0010) feedback model. Therefore, we show a new achievability based on block-Markov encoding and dirty paper encoding/decoding to achieve the rate-pair $(n, n-m)$ for the (0010) feedback model.

Encoding: The encoding strategy is shown in Table III. At T_1, there is no splitting of messages. At T_2, in the i^{th} block, the message is split into two parts $X_{2i,d}$ and $X_{2i,nd}$. Also, in the i^{th} block $X_{2i-1,d}$ is transmitted by T_2 such that it is decodable at D_2 right-away. The message $X_{2i-1,d}$ is known at T_1 before the i^{th} block of transmission due to the cross-link feedback. Therefore, T_1 can employ a dirty paper coding like strategy to allow its receiver to decode in the presence of interference $X_{2i-1,d}$ as shown in Table III.

TABLE III
ENCODING OF MESSAGES IN THE WEAK INTERFERENCE REGIME FOR THE (0010) FEEDBACK MODEL

	Block 1	Block i	Block B
Message X_{1i} at T_1	$[X_{11}]$	$[X_{1i} \oplus \mathbf{S}^{n-m} X_{2i-1}]$	$\mathbf{0}_n^T$
Message X_{2i} at T_2	$[\mathbf{0}_m^T, X_{21,nd}^T, X_{21,d}^T]^T$	$[X_{2i-1,d}^T, \mathbf{0}_p^T, X_{2i,nd}^T, X_{2i,d}^T]^T$	$[X_{2B-1,d}^T, \mathbf{0}_p^T, X_{2B,nd}^T, X_{2B,d}^T]^T$

Decoding: The messages X_{1i} are decodable at D_1 and $X_{2i-1,d}$ and $X_{2i,nd}$ are decodable at D_2 in the i^{th} block as long as the cardinality of the messages are

$$|X_{2i-1,d}| = \min(n-m, m), \ |X_{2i,nd}| = \max(n-2m, 0), \tag{32}$$

and $p = \max(2m-n, 0)$. As $B \to \infty$, the rate-pair $(n, n-m)$ is achievable, i.e., the corner point $\mathcal{K}_{\mathrm{B}}$ is achievable. In the weak interference regime, from Lemma 4.6 and the achievability of the (1000) feedback model, we know that in the weak interference regime, the corner point $\mathcal{K}_{\mathrm{D}} \equiv (n-m, n)$ is achievable with the (0010) feedback model.

In the strong interference regime, from Theorem 2.1, and the outer-bounds (13), (14) and (17), we know that $\mathcal{C}^{(0010)} \equiv \mathcal{C}^{(0000)}$. Thus, the capacity region characterization of the (0010) feedback model is complete.

C. Capacity Regions of the rest of the Feedback Models

For each feedback model, the capacity region is shown by the achievability of the subset of corner points (19) which form the tightest outer bound, among all the corner points.

1) $(\mathbf{1001}), (\mathbf{1101})$ *and* $(\mathbf{1111})$ *Feedback Models:* The capacity region of the (1001) feedback model is given in Theorem 2.2. It can also be derived using the outer-bounds given by (17), (13) and (14), and showing the achievability by treating the (1001) feedback model as a combination of the (1000) and (0001) feedback models. The outer-bound of the capacity region $\mathcal{C}^{(1001)}$ can be sufficiently characterized by the corner points $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$. We know that $\mathcal{K}_{\mathrm{D}}$ is achievable with (1000) feedback and thus by symmetry $\mathcal{K}_{\mathrm{B}}$ is achievable with (0001) feedback. Since $\mathcal{C}^{(1000)} \subseteq \mathcal{C}^{(1001)}$ and $\mathcal{C}^{(0001)} \subseteq \mathcal{C}^{(1001)}$, we conclude that $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$ are achievable with the (1001) feedback model.

The corner points $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$ also sufficiently characterize the outer-bound of the capacity region of the (1111) feedback model. As $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$ are both achievable with the (1001) feedback model,

$$\mathcal{C}^{(1001)} \equiv \mathcal{C}^{(1111)}. \tag{33}$$

To compare the (1xx1) feedback model with the (1000) feedback model, we note that the point $\mathcal{K}_{\mathrm{B}}$ is not achievable with the latter. This is because there is no virtual relay path available between T_1 and D_1. In the (1xx1) feedback model, a virtual relay route between T_1 and D_1 is available and therefore $\mathcal{K}_{\mathrm{B}}$ is achievable. Hence, $\mathcal{C}^{(1\mathrm{xx}1)} \supset \mathcal{C}^{(1000)}$, and the capacity region of the (1xx1) feedback model is strictly larger than the capacity region of the (1000) feedback model.

2) $(\mathbf{1100}), (\mathbf{1110})$ *and* $(\mathbf{1010})$ *Feedback Models:* We know that

$$\mathcal{C}^{(1100)} \subseteq \mathcal{C}^{(1110)} \subseteq \mathcal{C}^{(1111)}. \tag{34}$$

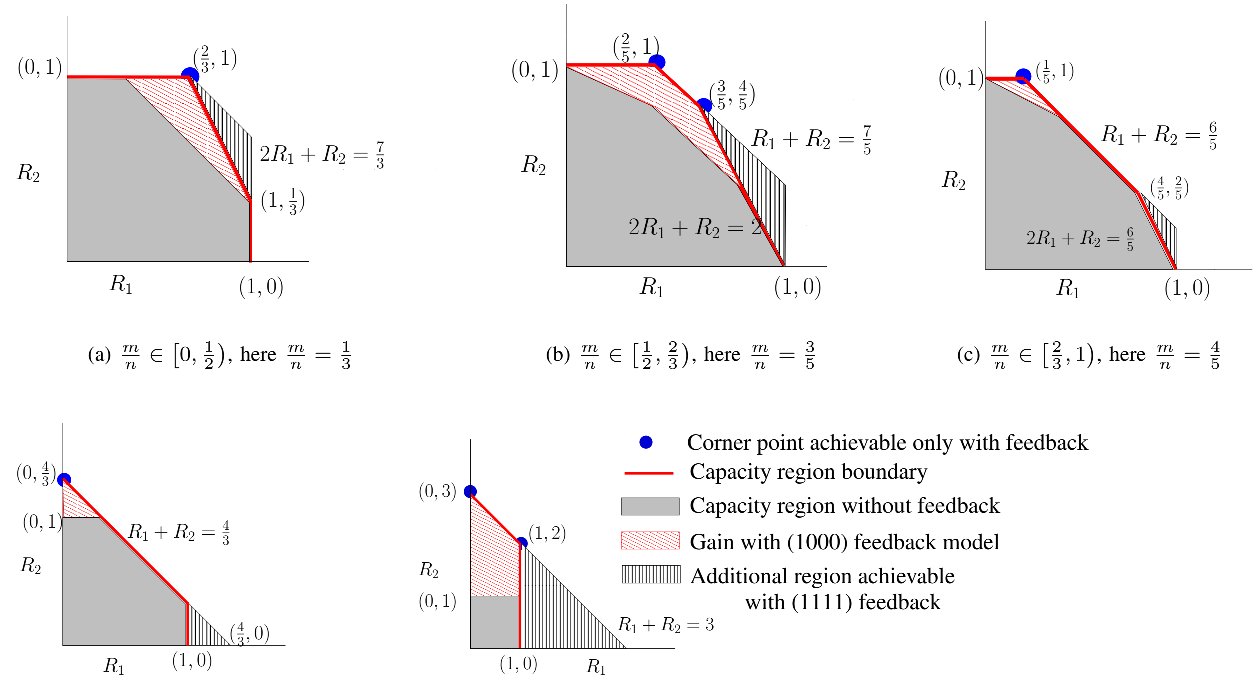
In the weak interference regime, where $n \geq m$, we know from Corollary 4.7 that $\mathcal{C}^{(1100)} \supseteq \mathcal{C}^{(1001)}$. Sandwiching the capacity regions $\mathcal{C}^{(1100)}$ and $\mathcal{C}^{(1110)}$ in between $\mathcal{C}^{(1001)}$ and $\mathcal{C}^{(1111)}$, from (33), (34) and (30), we conclude that in the weak interference regime

$$\mathcal{C}^{(1001)} \equiv \mathcal{C}^{(1100)} \equiv \mathcal{C}^{(1110)} \equiv \mathcal{C}^{(1111)}. \tag{35}$$

In the strong interference regime, where $n < m$, the corner points $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$ sufficiently characterize $\mathcal{C}^{(1100)}$. We know that when $n < m$, $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$ are also achievable with (1000) feedback. Since $\mathcal{C}^{(1110)} \supseteq \mathcal{C}^{(1100)} \supseteq \mathcal{C}^{(1000)}$, we can conclude that in the strong interference regime

$$\mathcal{C}^{(1110)} \equiv \mathcal{C}^{(1100)} \equiv \mathcal{C}^{(1000)}. \tag{36}$$

For the (1010) feedback model, the outer bound of the capacity region in the weak interference regime is characterized by $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$. The corner point $\mathcal{K}_{\mathrm{D}}$ is shown to be achievable with (1000) and corner point $\mathcal{K}_{\mathrm{B}}$ is



(a) $\frac{m}{n} \in [0, \frac{1}{2})$, here $\frac{m}{n} = \frac{1}{3}$

(b) $\frac{m}{n} \in [\frac{1}{2}, \frac{2}{3})$, here $\frac{m}{n} = \frac{3}{5}$

(c) $\frac{m}{n} \in [\frac{2}{3}, 1)$, here $\frac{m}{n} = \frac{4}{5}$

(d) $\frac{m}{n} \in (1, 2]$, here $\frac{m}{n} = \frac{4}{3}$

(e) $\frac{m}{n} \in (2, \infty)$, here $\frac{m}{n} = 3$

Fig. 6. Typical *normalized* capacity region poly-topes of the (1000) feedback model in different regimes of interference. The large dots represent the corner points *not* achievable without the (1000) feedback model. The figure also shows the capacity region of the interference channel with no feedback and with the (1111) feedback model.

achievable with (0010) feedback models. Thus, (1010) can achieve both corner points $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$. In the weak interference regime, since $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$ also characterize the outer-bound of the (1111) feedback model, in the weak interference regime we have

$$\mathcal{C}^{(1010)} \equiv \mathcal{C}^{(1111)}. \tag{37}$$

As $\mathcal{C}^{(1010)} \subseteq \mathcal{C}^{(1110)}$, from (36), we conclude that in the strong interference regime

$$\mathcal{C}^{(1010)} \equiv \mathcal{C}^{(1110)} \equiv \mathcal{C}^{(1100)} \equiv \mathcal{C}^{(1000)}. \tag{38}$$

3) $(\mathbf{0110})$ *Feedback Models:* The corner points $\mathcal{K}_{\mathrm{B}}$ and $\mathcal{K}_{\mathrm{D}}$ characterize the outer-bound for the (0110) feedback model as well as (0010), and since they are achievable with (0010), they are also achievable with the (0110) feedback model. Thus,

$$\mathcal{C}^{(0010)} \equiv \mathcal{C}^{(0110)}. \tag{39}$$

It is noteworthy that in the strong interference regime, from Theorem 2.1 and outer-bounds (13), (14) and (17), it can easily be confirmed that

$$\mathcal{C}^{(0000)} \equiv \mathcal{C}^{(0110)}. \tag{40}$$

D. Sum-capacity

1) Equivalence of the sum-capacity of all $(\mathbf{1}\mathrm{xxx})$ *Feedback Models:* In the achievability of the (1000) feedback model, we showed that in the weak interference regime, the rate pair $(n-m, n)$ and in the strong interference regime, the rate pair $(0, m)$ is achievable. These rate pairs $(n-m, n)$ and $(0, m)$ both lie on the outer-bound of the sum-capacity (17). Thus, using the achievability of the (1000) feedback model, we can show the achievability

of the rate pairs $(n-m,n)$ and $(0,m)$ for (1xxx) feedback models, which proves the result $C_{\text{sum}}^{(1\text{xxx})} = C_{\text{sum}}^{(1000)}$.

2) Sum-capacity of the $(\mathbf{0110})$ *and* $(\mathbf{0010})$ *Feedback Models:* From Lemma 4.4, in the weak interference regime $(n-m,n)$ lies on the outer-bound of the (1111) feedback model. Hence, it is sum-capacity achieving for (0010) as well as (0110) feedback models. From Lemma 4.6, in the weak interference regime any rate pair that is achievable with the (1000) feedback model should also be achievable with the (0010) feedback model. Since the rate pair $(n-m,n)$ is achievable with (1000), it is also achievable with (0010) and subsequently the (0110) feedback model. Therefore, in the weak interference regime $C_{\text{sum}}^{(0010)} = C_{\text{sum}}^{(0110)} = C_{\text{sum}}^{(1111)}$.

In the strong interference regime, we know from (39) and (40), that feedback does not improve the capacity region for the (0110) feedback model and therefore does not improve the capacity region of (0010) either. Thus, in the strong interference regime $C_{\text{sum}}^{(0010)} = C_{\text{sum}}^{(0110)} = C_{\text{sum}}^{(0000)}$.

V. Feedback Models: Gaussian channel

In this section, the approximate Gaussian capacity regions are derived for all 9 canonical feedback models. First, we derive two new outer bounds for the (1111) and (1000) feedback models. Then, we show an achievability based on Han-Kobayashi type message splitting for the (1000) model. We prove Lemma 5.4, which relates the achievable rate regions of the (0010) and (1000) feedback models. Additionally, we propose a block-Markov and dirty paper encoding based achievability scheme for the (0010) feedback model. Finally, using the achievability of the (1000) and (0010) feedback models, we show the approximate capacity regions for all canonical feedback models.

A. Outer Bounds for the Gaussian Channel

Now, we present two new outer bounds and the cut-set bound for the two-user interference channel with various feedback states.

Lemma 5.1 ([24, 25]): The two-user symmetric Gaussian interference channel with any one of the feedback models is outer bounded by

$$\begin{aligned} R_1 &\leq c_1 \qquad (41)\\ R_2 &\leq c_2, \qquad (42)\end{aligned}$$

where

$$c_1 = \begin{cases} \log(1+\mathsf{SNR}) & \text{if } \mathsf{T}_2 \text{ receives no direct-link feedback} \\ \log(1+\mathsf{SNR}+\mathsf{INR}) & \text{otherwise,} \end{cases} \qquad (43)$$

$$c_2 = \begin{cases} \log(1+\mathsf{SNR}) & \text{if } \mathsf{T}_1 \text{ receives no direct-link feedback} \\ \log(1+\mathsf{SNR}+\mathsf{INR}) & \text{otherwise.} \end{cases} \qquad (44)$$

The following theorem provides an outer-bound on the sum-capacity of the (1111) feedback model.

Theorem 5.2: The sum capacity of the two-user symmetric Gaussian interference channel for the (1111) feedback model is outer bounded by

$$R_1 + R_2 \leq \sup_{0\leq|\rho|\leq 1} \log\left(1 + \frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right) + \log(1+\mathsf{SNR}+\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}}). \qquad (45)$$

Proof: The proof details are provided in Appendix C. ■

Remark 4: As the sum-capacity of the (1111) feedback model is an outer bound on the sum-capacity of all feedback models, Theorem 5.2 also applies as an outer bound on the sum-capacity of all feedback models.
Note that (45) can further be upper bounded to yield

$$R_1 + R_2 \leq \log\left(1 + \frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1+\mathsf{SNR}+\mathsf{INR}+2\sqrt{\mathsf{SNR.INR}}). \qquad (46)$$

As observed in the deterministic case, the bound on the sum-capacity is not sufficient to describe the capacity region of the (1000) feedback model. The following theorem is an upper bound on the rate $2R_1 + R_2$.

Theorem 5.3: The capacity region of the two-user symmetric Gaussian interference channel with feedback state (1000) is outer bounded by

$$\begin{aligned} 2R_1 + R_2 \quad \leq \quad & \sup_{0\leq|\rho|\leq 1} \log\left(1 + \frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right) + \log(1+\mathsf{SNR}+\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}}) \\ & + \log\left(1+\mathsf{INR}+\frac{\mathsf{SNR}-(1+|\rho|^2)\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}}}{1+\mathsf{INR}}\right). \end{aligned} \tag{47}$$

Proof: The proof details are provided in Appendix D. ■

To characterize the approximate capacity region of the (1000) feedback model, we will use the bound in (47) only in the weak interference regime. In the weak interference regime, an upper bound for (47) is

$$2R_1 + R_2 \leq \log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1+\mathsf{SNR}+\mathsf{INR}+2\sqrt{\mathsf{SNR.INR}}) + \log\left(1+\mathsf{INR}+\frac{\mathsf{SNR}-\mathsf{INR}}{1+\mathsf{INR}}\right) \tag{48}$$

In Table IV, we present the approximate capacity regions of the different feedback models studied in this paper. The table also lists the gap to capacity for each of the feedback models. These gaps are computed for the achievability schemes that will be described in Section V-B.

TABLE IV
APPROXIMATE CAPACITY REGIONS FOR ALL 9 CANONICAL FEEDBACK MODELS

Cases	Outer bound of Capacity Region	Gap to Capacity
(1xx1)	$R_1 \leq \log(1+\mathsf{SNR}+\mathsf{INR})$ $R_2 \leq \log(1+\mathsf{SNR}+\mathsf{INR})$ $R_1+R_2 \leq \sup_{0\leq|\rho|\leq 1}\{\log\left(1+\frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right)+$ $\log(1+\mathsf{SNR}+\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}})\}$	2.59 bits/Hz
(1100), (1110) (1010)	$R_1 \leq \log(1+\mathsf{SNR})$ $R_2 \leq \log(1+\mathsf{SNR}+\mathsf{INR})$ $R_1+R_2 \leq \sup_{0\leq|\rho|\leq 1}\{\log\left(1+\frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right)+$ $\log(1+\mathsf{SNR}+\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}})\}$	2.59 bits/Hz for (1100) and (1110) 4.59 bits/Hz for (1010)
(0110), (0010)	$R_1 \leq \log(1+\mathsf{SNR})$ $R_2 \leq \log(1+\mathsf{SNR})$ $R_1+R_2 \leq \sup_{0\leq|\rho|\leq 1}\log\left(1+\frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right)+$ $\log(1+\mathsf{SNR}+\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}})$	2.59 bits/Hz for (0110) 4.59 bits/Hz for (0010)
(1000)	$R_1 \leq \log(1+\mathsf{SNR})$ $R_2 \leq \log(1+\mathsf{SNR}+\mathsf{INR})$ $R_1+R_2 \leq \sup_{0\leq|\rho|\leq 1}\{\log\left(1+\frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right)+$ $\log(1+\mathsf{SNR}+\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}})\}$ $2R_1+R_2 \leq \sup_{0\leq|\rho|\leq 1}\{\log\left(1+\frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right)+$ $\log(1+\mathsf{SNR}+\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}})+$ $\log\left(1+\mathsf{INR}+\frac{\mathsf{SNR}-(1+|\rho|^2)\mathsf{INR}+2|\rho|\sqrt{\mathsf{SNR.INR}}}{1+\mathsf{INR}}\right)\}$	4.59 bits/Hz

B. Achievability

In this section, we show the achievability of the sum-rate and the rate regions, which are within a constant number of bits of the outer bound developed in Section V-A. The achievable rate region as well as the outer-bound are implicitly parameterized by the pair $(\mathsf{SNR}, \mathsf{INR})$. Let the set of all corner points (vertices) of the convex polygon,

which forms the outer bound for feedback state $(F_{11}F_{12}F_{21}F_{22})$, be denoted by $\mathcal{Q}^{(F_{11}F_{12}F_{21}F_{22})}$. Then in order to prove that the capacity region is within a constant number of bits of the outer bound, it is sufficient to prove

$$\max_{\mathsf{SNR},\mathsf{INR}} \min_{\mathcal{R}^{(F_{11}F_{12}F_{21}F_{22})}} \max(\overline{C}_1 - R_1, \overline{C}_2 - R_2) \leq c, \tag{49}$$

where $(R_1, R_2) \in \mathcal{R}^{(F_{11}F_{12}F_{21}F_{22})}$, $(\overline{C}_1, \overline{C}_2) \in \mathcal{Q}^{(F_{11}F_{12}F_{21}F_{22})}$ and c is a fixed constant independent of SNR and INR. Therefore, in this section, for each corner point on the outer bound, we show an achievable rate pair that is within c bits from it. The corner points of relevance are defined here as

$$\begin{aligned}
\overline{\mathcal{K}}_{\mathrm{A}} &= \{(\overline{C}_1, \overline{C}_2) : \overline{C}_1 = R_1 \ \&\ \overline{C}_2 = R_2 \text{ such that (41) and (48) hold with equality simultaneously}\},\\
\overline{\mathcal{K}}_{\mathrm{B}} &= \{(\overline{C}_1, \overline{C}_2) : \overline{C}_1 = R_1 \ \&\ \overline{C}_2 = R_2 \text{ such that (41) and (46) hold with equality simultaneously}\},\\
\overline{\mathcal{K}}_{\mathrm{C}} &= \{(\overline{C}_1, \overline{C}_2) : \overline{C}_1 = R_1 \ \&\ \overline{C}_2 = R_2 \text{ such that (46) and (48) hold with equality simultaneously}\},\\
\overline{\mathcal{K}}_{\mathrm{D}} &= \{(\overline{C}_1, \overline{C}_2) : \overline{C}_1 = R_1 \ \&\ \overline{C}_2 = R_2 \text{ such that (42) and (46) hold with equality simultaneously}\},\\
\overline{\mathcal{K}}_{\mathrm{E}} &= \{(\overline{C}_1, \overline{C}_2) : \overline{C}_1 = R_1 \ \&\ \overline{C}_2 = R_2 \text{ such that (41) and (42) hold with equality simultaneously}\}.
\end{aligned} \tag{50}$$

Note that $\overline{\mathcal{K}}_{\mathrm{A}}$ and $\overline{\mathcal{K}}_{\mathrm{C}}$ are defined only for the weak interference regime as we will need to show achievable rate pairs within constant number of bits from them only in the weak interference regime. Moreover, note that for a fixed $\mathsf{SNR}, \mathsf{INR}$ the rate pair described by a corner point in the outer bound will change based on the feedback model, since the bounds (41) and (42) vary based on the feedback model.

We next describe the achievability for the (1000) and (0010) feedback models, find $\mathcal{R}^{(1000)}$ and $\mathcal{R}^{(0010)}$, and then use them to obtain the approximate capacity regions of all 9 canonical feedback models.

1) Achievability for the $(\mathbf{1000})$ *Feedback Model:*

a) **Weak Interference***,* $\alpha \leq 1$*:* The outer-bound of the capacity region of the (1000) feedback model is sufficiently characterized by $\overline{\mathcal{K}}_{\mathrm{A}}$, $\overline{\mathcal{K}}_{\mathrm{C}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$. To achieve within constant number of bits of $\overline{\mathcal{K}}_{\mathrm{A}}$, feedback is not required, while to achieve within a constant number bits of $\overline{\mathcal{K}}_{\mathrm{C}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$, feedback is needed.

Encoding: Similar to the achievability in Section IV, we use the Han-Kobayashi rate-splitting approach [23]. At both transmitters the message to be transmitted is split into common and private parts. The common message generated by T_2 in the i^{th} block is learned by T_1 via feedback, decoded, re-encoded and re-transmitted in the $(i+1)^{\mathrm{th}}$ block. In the i^{th} block of transmission, the common and private messages generated by the u^{th} transmitter are denoted by $X_{ui,c}$ and $X_{ui,p}$ respectively. Rates R_{up}, R_{uc} denote the private and common rates for the $\mathsf{T}_u - \mathsf{D}_u$ pair. Thus, $R_u = R_{up} + R_{uc}$. The fraction of power allocated to the common and private parts is λ_{uc} and λ_{up}. To transmit the common message of T_2, T_1 allocates λ_{1r} fraction of its power. The power constraint implies $\lambda_{1c} + \lambda_{1p} + \lambda_{1r} \leq 1$ and $\lambda_{2c} + \lambda_{2p} \leq 1$. As a simplification step, we propose $\lambda_{1p} = \lambda_{2p}$. The following communication strategy, which extends to B blocks is proposed

$$X_{1i} = \begin{cases} 0 & i = 1 \\ \sqrt{\lambda_{1p}}X_{1i,p} + \sqrt{\lambda_{1c}}X_{1i,c} + \sqrt{\lambda_{1r}}X_{2i-1,c} & 1 < i \leq B \end{cases} \tag{51}$$

and

$$X_{2i} = \begin{cases} \sqrt{\lambda_{2p}}X_{2i,p} + \sqrt{\lambda_{2c}}X_{2i,c} & 1 \leq i < B \\ 0 & i = B \end{cases} \tag{52}$$

Decoding: We will employ *forward decoding* at D_2 and *backward decoding* (starting from the B^{th} block) at D_1. Since forward decoding is used at D_2, the message $X_{2i-1,c}$ is decoded before decoding the i^{th} block. Thus, $g_c\sqrt{\lambda_{1r}}X_{2i-1,c}$ can be subtracted from the received message Y_{2i} while decoding the messages received in the i^{th} block. On the other hand, at D_1, since backward decoding is employed, message $X_{2i,c}$ is decoded while decoding the $(i+1)^{\mathrm{th}}$ block of received messages. Thus, it can be used to subtract out $g_c\sqrt{\lambda_{2c}}X_{2i,c}$ from the received message Y_{1i} to assist decoding the i^{th} block. At D_1, the private messages $X_{1i,p}$ and $X_{2i,p}$ are treated as noise while decoding $X_{1i,c}$ and $X_{2i-1,c}$. After decoding $X_{1i,c}$ and $X_{2i-1,c}$, they are subtracted out from Y_{1i} and $X_{1i,p}$ is decoded treating $X_{2i,p}$ as noise. Similar steps follow at the receiver D_2. At D_1, the decoding constraint can be

TABLE V
POWER ALLOCATION FOR THE PRIVATE AND COMMON MESSAGES FOR (1000) FEEDBACK MODEL

Corner Point	α	λ_{1p}	λ_{2p}	λ_{1c}	λ_{2c}	λ_{1r}
$\mathcal{P}_{\rm A}$	$[0, 1/2)$	1	$\min(1, 1/\mathsf{INR})$	0	0	0
	$[1/2, 1]$	1	0	0	0	0
$\mathcal{P}_{\rm B}$	$(1, 2]$	0	0	1	1	0
	$(2, \infty)$	0	0	$1 - \frac{1}{\mathsf{SNR}}$	1	$\frac{1}{\mathsf{SNR}}$
$\mathcal{P}_{\rm C}$	$[0, 1/2)$	$\min(1, 1/\mathsf{INR})$	$\min(1, 1/\mathsf{INR})$	0	$1 - \lambda_{1p}$	$1 - \lambda_{2p}$
	$[1/2, 2/3)$	$\min(1, 1/\mathsf{INR})$	$\min(1, 1/\mathsf{INR})$	$\frac{(1-\lambda_{1p})}{2}$	$(1 - \lambda_{2p})$	$\frac{(1-\lambda_{1p})}{2}$
	$[2/3, 1]$	$\min(1, 1/\mathsf{INR})$	$\min(1, 1/\mathsf{INR})$	1 - λ_{1p}	1 - λ_{2p}	0
$\mathcal{P}_{\rm D}$	$[0, 1]$	$\min(1, 1/\mathsf{INR})$	$\min(1, 1/\mathsf{INR})$	0	$1 - \lambda_{1p}$	$1 - \lambda_{2p}$
	$(1, \infty)$	0	0	0	1	1

written as

$$R_{1c} \leq \log\left(1 + \frac{\lambda_{1c}\mathsf{SNR}}{\lambda_{1p}\mathsf{SNR} + \lambda_{2p}\mathsf{INR} + 1}\right) \tag{53}$$

$$R_{2c} \leq \log\left(1 + \frac{\lambda_{1r}\mathsf{SNR}}{\lambda_{1p}\mathsf{SNR} + \lambda_{2p}\mathsf{INR} + 1}\right) \tag{54}$$

$$R_{1c} + R_{2c} \leq \log\left(1 + \frac{(\lambda_{1c} + \lambda_{1r})\mathsf{SNR}}{\lambda_{1p}\mathsf{SNR} + \lambda_{2p}\mathsf{INR} + 1}\right), \tag{55}$$

while at D_2, the decoding constraints are

$$R_{1c} \leq \log\left(1 + \frac{\lambda_{1c}\mathsf{INR}}{\lambda_{1p}\mathsf{SNR} + \lambda_{2p}\mathsf{INR} + 1}\right) \tag{56}$$

$$R_{2c} \leq \log\left(1 + \frac{\lambda_{2c}\mathsf{SNR}}{\lambda_{1p}\mathsf{SNR} + \lambda_{2p}\mathsf{INR} + 1}\right) \tag{57}$$

$$R_{1c} + R_{2c} \leq \log\left(1 + \frac{\lambda_{1c}\mathsf{INR} + \lambda_{2c}\mathsf{SNR}}{\lambda_{1p}\mathsf{SNR} + \lambda_{2p}\mathsf{INR} + 1}\right). \tag{58}$$

Further, since we are employing a decode and forward kind of strategy for re-transmitting $X_{2i-1,c}$, before forwarding it, T_1 has to decode it using the signal $(Y_{1i} - g_d X_{1i})$ (X_{1i} available via feedback). This imposes the following decoding constraints

$$R_{2c} \leq \log\left(1 + \frac{\lambda_{2c}\mathsf{INR}}{\lambda_{2p}\mathsf{INR} + 1}\right), \tag{59}$$

Finally, the decoding constraints for the private messages are

$$R_{1p} \leq \log\left(1 + \frac{\lambda_{1p}\mathsf{SNR}}{\lambda_{2p}\mathsf{INR} + 1}\right) \tag{60}$$

$$R_{2p} \leq \log\left(1 + \frac{\lambda_{2p}\mathsf{SNR}}{\lambda_{1p}\mathsf{INR} + 1}\right). \tag{61}$$

Choice of power and rate allocation: In Tables V and VI, the power and corresponding rate allocation for four different rate pairs (R_1, R_2), labeled $\mathcal{P}_{\rm A}, \mathcal{P}_{\rm B}, \mathcal{P}_{\rm C}$ and $\mathcal{P}_{\rm D}$ are shown. Note that, for each of the rate pairs labeled by $\mathcal{P}_{\rm A}, \mathcal{P}_{\rm B}, \mathcal{P}_{\rm C}$ and $\mathcal{P}_{\rm D}$, we can obtain $R_1 = R_{1p} + R_{1c}$ and $R_2 = R_{2p} + R_{2c}$ from Table VI. Using the achievable strategy for the (1000) feedback model, in the weak interference regime, the rate pairs labeled by $\mathcal{P}_{\rm A}, \mathcal{P}_{\rm C}$ and $\mathcal{P}_{\rm D}$, described in Table VI, are easily shown to be feasible for the power allocation described in Table V.

The rate pairs described by $\mathcal{P}_{\rm A}$, $\mathcal{P}_{\rm C}$ and $\mathcal{P}_{\rm D}$ in Table VI are within a constant number of bits from $\overline{\mathcal{K}}_{\rm A}$, $\overline{\mathcal{K}}_{\rm C}$ and $\overline{\mathcal{K}}_{\rm D}$ respectively. The gaps of $\mathcal{P}_{\rm A}$, $\mathcal{P}_{\rm C}$ and $\mathcal{P}_{\rm D}$ from $\overline{\mathcal{K}}_{\rm A}$, $\overline{\mathcal{K}}_{\rm C}$ and $\overline{\mathcal{K}}_{\rm D}$ for the (1000) feedback model are evaluated in Appendix E1, E2 and E3 and the maximum gap is found to be 4.59 bits/Hz.

TABLE VI
RATE ALLOCATION TO THE PRIVATE AND COMMON MESSAGES FOR (1000) FEEDBACK MODEL

Corner Point	α	R_{1p}	R_{2p}	R_{1c}	R_{2c}
$\mathcal{P}_{\mathrm{A}}$	$[0, 1/2)$	$\log(\mathsf{SNR}/2)$	$\log(\mathsf{SNR}/2\mathsf{INR}^2)$	0	0
	$[1/2, 1]$	$\log(1+\mathsf{SNR})$	0	0	0
$\mathcal{P}_{\mathrm{B}}$	$(1, 2]$	0	0	$\log(\mathsf{SNR})$	$\log(1+\frac{\mathsf{INR}}{\mathsf{SNR}})$
	$(2,\infty)$	0	0	$\log(\mathsf{SNR})$	$\log(\frac{\mathsf{INR}}{\mathsf{SNR}})$
$\mathcal{P}_{\mathrm{C}}$	$[0, 1/2)$	$\log(1+\mathsf{SNR}/2\mathsf{INR})$	R_{1p}	0	$\log(\mathsf{INR}/3)$
	$[1/2, 2/3)$	$\log(1+\frac{\mathsf{SNR}}{2\mathsf{INR}})$	R_{1p}	$\log(1+\frac{\mathsf{INR}^2}{\mathsf{SNR}})-2$	$\log(\frac{1+\mathsf{SNR}/\mathsf{INR}}{4})$
	$[2/3, 1]$	$\log(1+\mathsf{SNR}/2\mathsf{INR})$	R_{1p}	$\log(\mathsf{INR}^2/3\mathsf{SNR})$	$\log(2\mathsf{SNR}/3\mathsf{INR})$
$\mathcal{P}_{\mathrm{D}}$	$[0, 1]$	$\log(1+\frac{\mathsf{SNR}}{2\mathsf{INR}})$	R_{1p}	0	$\log(\frac{\mathsf{INR}}{3})$
	$(1,\infty)$	0	0	0	$\log(1+\mathsf{INR})$

b) **Strong Interference***, $\alpha > 1$:* The outer-bound is sufficiently described by $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$. For $1 < \alpha < 2$, the achievable rate pair described by $\mathcal{P}_{\mathrm{B}}$ in Table VI can be achieved without feedback and is within constant number of bits from $\overline{\mathcal{K}}_{\mathrm{B}}$. The constant is evaluated to be 2.59 bits/Hz in Appendix E6. For the rest, the following achievable strategy is employed.

The encoding is identical with (51) and (52). In contrast to the decoding scheme for $\alpha \leq 1$, in strong interference, we employ forward decoding at D_1 and backward decoding at D_2. Private messages are not needed in this regime, thus $\lambda_{1p} = \lambda_{2p} = 0$, and correspondingly $R_{1p} = R_{2p} = 0$. Since forward decoding is employed at D_1, $X_{2i-1,c}$ is decoded from the received message in the $(i-1)^{\mathrm{th}}$ block of decoding, and thus $g_d\sqrt{\lambda_{1r}}X_{2i-1,c}$ can be subtracted out from the received message Y_{1i} for decoding the i^{th} block. On the other hand, at D_2, backward decoding is applied. Thus, prior to decoding the i^{th} block, $X_{2i,c}$ is known and can be used to subtract $g_d\sqrt{\lambda_{2c}}X_{2i,c}$ from Y_{2i}. Then, $X_{2i-1,c}$ and $X_{1i,c}$ are decoded. With $\lambda_{up} = 0$, the decoding constraints at D_1, D_2 and T_1 are the same as (53)-(55), (56)-(58) and (59).

Choice of power and rate allocation: In the strong interference regime, rate pairs described by $\mathcal{P}_{\mathrm{B}}$ and $\mathcal{P}_{\mathrm{D}}$ in Table VI are feasible for the power allocation described by Table V. The rate pairs described by $\mathcal{P}_{\mathrm{B}}$ and $\mathcal{P}_{\mathrm{D}}$ in Table VI are within constant number of bits of $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$ respectively, for which the gap is computed in Appendix E6 and E7. The maximum gap is found to be 2.59 bits/Hz. This completes the characterization of the approximate capacity for the (1000) feedback within 4.59 bits/Hz.

2) Relating $\mathcal{R}^{(1000)}$ and $\mathcal{R}^{(0010)}$: For proving the achievability of the rest of the feedback models, we prove the following lemma, which relates $\mathcal{R}^{(1000)}$ and $\mathcal{R}^{(0010)}$.

Lemma 5.4: When $\alpha \leq 1$, there exist achievable rate regions $\mathcal{R}^{(0010)}$ and $\mathcal{R}^{(0100)}$ such that

$$\mathcal{R}^{(1000)} \subseteq \mathcal{R}^{(0010)} \tag{62}$$

$$\mathcal{R}^{(0001)} \subseteq \mathcal{R}^{(0100)}, \tag{63}$$

where the region $\mathcal{R}^{(1000)}$ is described for the feedback model (1000) in Section V-B1. The region $\mathcal{R}^{(0001)}$ is such that if $(R_x, R_y) \in \mathcal{R}^{(1000)}$, then $(R_y, R_x) \in \mathcal{R}^{(0001)}$.

Proof: Due to the symmetry, proving (62) is sufficient. Suppose that for the (0010) feedback model, the encoding is identical to the one in the (1000) feedback model. Then, the feasibility of decoding needs to be established for the (0010) feedback model, given that decoding is feasible for the (1000) feedback model. Let the decoding at both the receivers also be identical. Then for a given choice of $\{R_{1c}, R_{2c}, R_{1p}, R_{2p}\}$ and $\{\lambda_{1c}, \lambda_{2c}, \lambda_{1p}, \lambda_{2p}, \lambda_{2r}\}$, the decoding constraints at the receivers for the (0010) feedback model are identical to (53)-(58) and (60),(61), which are known to be feasible for the (1000) feedback model. The decoding constraints at T_1 are different in (0010) compared to (1000), since the feedback messages are different. Since T_1 knows its own transmitted symbol X_{1i}, the common message $X_{2i,c}$ needs to be decoded from $Y_{2i} - g_c X_{1i}$, for which the decoding constraint is

$$R_{2c} \leq \log\left(1 + \frac{\lambda_{2c}\mathsf{SNR}}{\lambda_{2p}\mathsf{INR}+1}\right). \tag{64}$$

Since $\alpha \leq 1$, i.e., $\mathsf{SNR} \geq \mathsf{INR}$, if a rate R_{2c} satisfies the constraint (59), then it also satisfies the constraint (64). Thus $\mathcal{R}^{(0010)}$ is achievable if $\mathcal{R}^{(1000)}$ is achievable, and the proof is complete.

■

TABLE VII
POWER ALLOCATION FOR THE (0010) FEEDBACK MODEL

Corner Point	α	λ_1	$\lambda_{2,d}$	$\lambda_{2,nd}$
$\mathcal{P}_{\mathrm{B2}}$	$[0,1/2)$	1	$1-1/\mathsf{INR}$	$1/\mathsf{INR}-\mathsf{SNR}/\mathsf{INR}$
	$[1/2,1]$		$1-1/\mathsf{INR}$	0

Corner Point	α	λ_{1p}	λ_{2p}	λ_{1c}	λ_{2c}
$\mathcal{P}_{\mathrm{B2}}$	$(1,2)$	0	0	1	$\mathsf{INR}/\mathsf{SNR}^2$
$\mathcal{P}_{\mathrm{D2}}$	$(1,2)$	0	0	$\mathsf{INR}/\mathsf{SNR}^2$	1
$\mathcal{P}_{\mathrm{E}}$	$[2,\infty)$	0	0	1	1

TABLE VIII
RATE ALLOCATION FOR THE (0010) FEEDBACK MODEL

Corner Point	α	R_1	$R_{2,d}$	$R_{2,nd}$
$\mathcal{P}_{\mathrm{B2}}$	$[0,1/2)$	$\log(1+\mathsf{SNR}/2)$	$\log(\mathsf{INR}/2)$	$\log(\mathsf{SNR}/\mathsf{INR}^2)$
	$[1/2,1]$		$1-1/\mathsf{INR})$	0

Corner Point	α	R_{1p}	R_{2p}	R_{1c}	R_{2c}
$\mathcal{P}_{\mathrm{B2}}$	$(1,2)$	0	0	$\log(\mathsf{INR})$	$\log(1+\mathsf{INR}/\mathsf{SNR})$
$\mathcal{P}_{\mathrm{D2}}$	$(1,2)$	0	0	$\log(1+\mathsf{INR}/\mathsf{SNR})$	$\log(\mathsf{INR})$
$\mathcal{P}_{\mathrm{E}}$	$[2,\infty)$	0	0	$\log(1+\mathsf{SNR})$	$\log(1+\mathsf{SNR})$

3) Achievability for the $(\mathbf{0010})$ *Feedback Model:*

a) **Weak Interference***,* $\alpha \leq 1$*:* The corner points $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$ characterize the outer-bound. From Lemma 5.4 and achievability of the (1000) feedback model, we know that $\mathcal{P}_{\mathrm{D}}$, described in Table VI is achievable. The gap of $\mathcal{P}_{\mathrm{D}}$ from $\overline{\mathcal{K}}_{\mathrm{D}}$ for (0010) feedback model is evaluated in Appendix E4 and is found to be 2.59 bits/Hz. However, to achieve within a constant number of bits of $\overline{\mathcal{K}}_{\mathrm{B}}$, we propose a different achievable scheme based on block-Markov encoding at T_2 and dirty paper encoding at T_1.

Encoding: At T_2, the message is split into two parts $X_{2i,d}$ and $X_{2i,nd}$ with rates $R_{2,d}$ and $R_{2,nd}$, such that $R_{2,d}+R_{2,nd}=R_2$. The transmitted message in the i^{th} block is

$$X_{2i}=\sqrt{\lambda_{2,d}}X_{2i-1,d}+\sqrt{\lambda_{2,nd}}X_{2i,nd}+\sqrt{1-\lambda_{2,d}-\lambda_{2,nd}}X_{2i,d} \tag{65}$$

such that the power constraint is $\lambda_{2,nd}+\lambda_{2,d}\leq 1$. At T_1, assuming that $X_{2i-1,d}$ can be decoded from the cross-link feedback, before the i^{th} block of transmission, the message to be transmitted is encoded into X_{1i} using dirty paper coding, treating $g_c\sqrt{\lambda_{2,d}}X_{2i-1,d}$ as interference. The encoded message is denoted as X_{1i}, and its rate is denoted by R_1.

Decoding: At D_2, backward decoding is applied. In the $(i+1)^{\text{th}}$ block, $X_{2i,d}$ and $X_{2i+1,nd}$ are assumed to be decoded. To decode $X_{2i-1,d}$ and $X_{2i,nd}$ from the i^{th} block, $g_d\sqrt{1-\lambda_{2,d}-\lambda_{2,nd}}X_{2i,d}$ is subtracted from Y_{2i} and X_{1i} is treated as noise. At D_1, dirty paper decoding is performed to decode the message from T_1 assuming $X_{2i,nd}$ and $X_{2i,d}$ as noise. At T_1, after the i^{th} transmission block Y_{2i} is received from the cross-link feedback from D_2. Since $X_{2i-1,d}$ is assumed to be known at T_1 before the i^{th} block, from Y_{2i}, $g_cX_{1i}+g_d\sqrt{\lambda_{2,d}}X_{2i-1,d}$ is subtracted to decode $X_{2i,nd}$ and $X_{2i,d}$.

Choice of power and rate allocation: Using the above encoding and decoding strategy, power and rate allocation for the rate pair described by $\mathcal{P}_{\mathrm{B2}}$ in Table VII and VIII is feasible in the weak interference regime. The gap of $\mathcal{P}_{\mathrm{B2}}$ from $\overline{\mathcal{K}}_{\mathrm{B}}$ is computed in Appendix E5 and the gap is found to be 4.59 bits/Hz.

b) **Strong Interference***,* $\alpha > 1$*:* In this regime of interference, the approximate capacity region can be achieved without feedback. When $1<\alpha<2$, the corner points $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$ characterize the outer bound of the capacity region. We have shown the power and rate allocation for rate pairs described by $\mathcal{P}_{\mathrm{B2}}$ and $\mathcal{P}_{\mathrm{D2}}$ in Table VII and VIII, which can be achieved without any feedback [9]. The gaps of $\mathcal{P}_{\mathrm{B2}}$ and $\mathcal{P}_{\mathrm{D2}}$ from $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$ respectively are both computed to be 2 bits/Hz. For $\alpha\geq 2$, $\overline{\mathcal{K}}_{\mathrm{E}}$ is the only non-trivial corner point on the outer bound. As the interference is strong enough, it can be completely decoded. Therefore, the channel is equivalent to two parallel point to point channels. Thus the rate pair described by $\mathcal{P}_{\mathrm{E}}$ in Tables VII and VIII is achievable. The gap of $\mathcal{P}_{\mathrm{E}}$ from $\overline{\mathcal{K}}_{\mathrm{E}}$ is 0. Thus, the achievability of (0010) feedback model within 4.59 bits/Hz is complete.

4) Achievability for the $(\mathbf{1001})$*,* $(\mathbf{1101})$ *and* $(\mathbf{1111})$ *Feedback Models:* We will find an achievable rate region for (1001), which is within a constant number of bits away from the outer-bound of the (1111) feedback model. For (1111), (1101) and (1001) feedback models, $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$ sufficiently characterize the outer-bound. Using (1000) feedback model, the rate pair described by $\mathcal{P}_{\mathrm{D}}$ in Table VI is achievable, and thus $\mathcal{P}_{\mathrm{D}}$ in Table VI is achievable with (1111), (1101) and (1001) feedback models. The gap of $\mathcal{P}_{\mathrm{D}}$ from $\overline{\mathcal{K}}_{\mathrm{D}}$ is evaluated in Appendix E3 and E7 to be 2.59 bits/Hz. From symmetry, (0001) feedback model can achieve a rate-pair within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$. As, $\mathcal{R}^{(1001)} \supseteq \mathcal{R}^{(1000)}$, and $\mathcal{R}^{(1001)} \supseteq \mathcal{R}^{(0001)}$, $\mathcal{R}^{(1001)}$ contains achievable rate pairs within 2.59 bits/Hz from both $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$. Also, as the following relation holds

$$\mathcal{R}^{(1001)} \subseteq \mathcal{C}^{(1001)} \subseteq \mathcal{C}^{(1101)} \subseteq \mathcal{C}^{(1111)} \tag{66}$$

and since $\mathcal{R}^{(1001)}$ is within 2.59 bits/Hz of the outer bound on the (1111) feedback model, we conclude that $\mathcal{C}^{(1001)}$ is within 2.59 bits/Hz of $\mathcal{C}^{(1111)}$. Since the achievability of $\mathcal{R}^{(1001)}$ directly follows from the achievability of $\mathcal{R}^{(1000)}$, the approximate capacity region characterization of all the feedback models of type $(1\text{xx}1)$ is complete.

5) Achievability for the $(\mathbf{1100})$*,* $(\mathbf{1110})$ *and* $(\mathbf{1010})$ *Feedback Models:* As more feedback can only increase the capacity region, we have

$$\mathcal{C}^{(1100)} \subseteq \mathcal{C}^{(1110)}. \tag{67}$$

The outer bound of the (1110) feedback model is characterized by the corner points $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$. Note that in the previous subsection, it is proved that the rate pair $\mathcal{P}_{\mathrm{D}}$, which is described by Table VI, is within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{D}}$. As the achievable rate region $\mathcal{R}^{(1000)}$ contains $\mathcal{P}_{\mathrm{D}}$ and

$$\mathcal{R}^{(1000)} \subseteq \mathcal{R}^{(1100)} \subseteq \mathcal{R}^{(1110)} \text{ and } \mathcal{R}^{(1000)} \subseteq \mathcal{R}^{(1010)}, \tag{68}$$

(1100), (1110) and (1010) feedback models also contain $\mathcal{P}_{\mathrm{D}}$, which is within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{D}}$. Now, we show the achievability of a rate pair within a constant number of bits from $\overline{\mathcal{K}}_{\mathrm{B}}$ for (1100), (1110) and (1010) feedback models.

a) **Weak Interference***,* $\alpha \leq 1$*:* In this regime, from Lemma 5.4, we know that $\mathcal{R}^{(0100)} \supseteq \mathcal{R}^{(0001)}$. As we have $\mathcal{R}^{(0110)} \supseteq \mathcal{R}^{(0100)}$, then $\mathcal{R}^{(0110)} \supseteq \mathcal{R}^{(0100)} \supseteq \mathcal{R}^{(0001)}$. From symmetry, we know that the achievable rate region $\mathcal{R}^{(0001)}$ contains a rate pair within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$. Thus, the rate regions $\mathcal{R}^{(0100)}$ and $\mathcal{R}^{(0110)}$ and subsequently $\mathcal{R}^{(1100)}$ and $\mathcal{R}^{(1110)}$ contain a rate pair within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$.

The (1010) feedback model can achieve any rate pair, which the (0010) feedback model can achieve. Since the rate pair described by $\mathcal{P}_{\mathrm{B2}}$ in Table VIII is achievable within 4.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$ for (0010) feedback model, thus $\mathcal{P}_{\mathrm{B2}}$ is also achievable with (1010) feedback model and is within 4.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$.

b) **Strong Interference***,* $\alpha > 1$*:* In this regime, recall that the achievable rate region $\mathcal{R}^{(1000)}$ itself contains the rate pair $\mathcal{P}_{\mathrm{B}}$, described in Table VI, which is within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$. Thus, $\mathcal{R}^{(1100)}$, $\mathcal{R}^{(1010)}$ and $\mathcal{R}^{(1110)}$ also contain a rate pair within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$.

6) Achievability for the $(\mathbf{0110})$ *Feedback Model:*

a) **Weak Interference***,* $\alpha \leq 1$*:* In this regime of interference, the outer bound on the capacity region of the (0110) feedback model is characterized by the corner points $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$. When $\alpha \leq 1$, due to Lemma 5.4 we know that $\mathcal{R}^{(1000)} \subseteq \mathcal{R}^{(0010)}$ and $\mathcal{R}^{(0001)} \subseteq \mathcal{R}^{(0100)}$. Also, $\mathcal{R}^{(1000)}$ contains the rate pair $\mathcal{P}_{\mathrm{D}}$, described in Table VI, which is within 2.59 bits/Hz of $\overline{\mathcal{K}}_{\mathrm{D}}$ as shown in Appendix E4. Symmetrically a rate pair is achievable within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{B}}$. Thus, $\mathcal{R}^{(0010)}$ and $\mathcal{R}^{(0100)}$ also contain rate pairs within 2.59 bits/Hz from $\overline{\mathcal{K}}_{\mathrm{D}}$ and $\overline{\mathcal{K}}_{\mathrm{B}}$. Consequently $\mathcal{R}^{(0110)}$ includes a rate pair, which is within 2.59 bits/Hz of both $\overline{\mathcal{K}}_{\mathrm{D}}$ and $\overline{\mathcal{K}}_{\mathrm{B}}$.

b) **Strong Interference***,* $\alpha > 1$*:* The rate pair achievable by the (0010) feedback model is also achievable by the (0110) feedback model. The corner points $\overline{\mathcal{K}}_{\mathrm{B}}$ and $\overline{\mathcal{K}}_{\mathrm{D}}$, which characterize the outer bound of the capacity region when $1 < \alpha < 2$, are both achievable within a constant number of bits without feedback by the rate pairs described by $\mathcal{P}_{\mathrm{B2}}$ and $\mathcal{P}_{\mathrm{D2}}$ in Table VIII. The gap of $\mathcal{P}_{\mathrm{D2}}$ from $\overline{\mathcal{K}}_{\mathrm{D}}$ is computed in Appendix E8 and is found to be 2 bits/Hz. Due to symmetry, the gap of $\mathcal{P}_{\mathrm{B2}}$ from $\overline{\mathcal{K}}_{\mathrm{B}}$ is also 2 bits/Hz. When $\alpha > 2$, the only non-trivial corner point on the outer bound is $\overline{\mathcal{K}}_{\mathrm{E}}$, which is achievable without any feedback with 0 gap from the rate pair $\mathcal{P}_{\mathrm{E}}$ described in Table VIII.

7) Sum-capacity of all feedback models: In order to characterize the sum-capacity of all feedback models, with at least one feedback link, in the weak interference regime, we use the outer bound on the (1111) feedback model. In the weak interference regime, the corner point $\overline{\mathcal{K}}_{\rm D}$ for (1111) feedback model is outside the capacity region of all feedback models. We know that the corner point $\mathcal{P}_{\rm D}$, described by Table VI is achievable for all feedback models in the weak interference regime. The distance of the sum-rate described by $\mathcal{P}_{\rm D}$ from $\overline{\mathcal{K}}_{\rm D}$ for the (1111) feedback model can be computed from Appendix E3 and is found to be 4.59 bits/Hz. Thus, all feedback models can achieve a rate pair within 4.59 bits/Hz from $\overline{\mathcal{K}}_{\rm D}$. Since $\overline{\mathcal{K}}_{\rm D}$ for the (1111) feedback model lies on the sum-rate outer bound on the (1111) feedback models, thus it lies on the sum-rate outer bound of all feedback models. For all feedback models, we have shown an achievable rate pair, $\mathcal{P}_{\rm D}$, which is within 4.59 bits/Hz from the sum-rate outer bound of (1111) feedback model. Thus, the sum-capacity of all feedback models, in the weak interference regime, is within 4.59 bits/Hz of each other.

8) Sum-capacity of (1xxx) *feedback models:* In the strong interference regime, all feedback models of type (1xxx), can achieve a rate pair described by $\mathcal{P}_{\rm D}$, which is within 3 bits/Hz from the corner point $\overline{\mathcal{K}}_{\rm D}$ that lies on sum-rate outer bound of the (1111) feedback model as shown in Appendix E7. Thus, in strong interference regime, all feedback models of type (1xxx), in the strong interference regime, is within 3 bits/Hz of each other.

VI. CONCLUSION

In this paper, we characterize the capacity region of all channel output feedback models in a two user symmetric interference channel. Depending on whether an infinite capacity feedback link exists between a receiver and a transmitter, a total of 9 canonical feedback models are present. In case of the symmetric linear deterministic interference channel, we find the exact capacity region, while for the Gaussian channel we find the approximate capacity region within at most 4.59 bits/Hz for all the 9 feedback models. Interestingly, in the weak interference regime all models of feedback have the identical capacity region except the feedback model with a single direct feedback link. In other words, all feedback models (except the single direct link feedback model) have the same capacity region as the capacity region achievable with all four feedback links. In particular, this includes that the capacity region of the single cross link feedback model is identical with the capacity region of the feedback model with all four feedback links. Although the single direct-link feedback has a smaller capacity region than other feedback models, in the weak interference regime, its sum-capacity is identical to the sum-capacity of the rest of the feedback models. In the strong interference regime as well, single direct-link feedback is sufficient to achieve the same sum-capacity as that achievable with all four feedback links.

To prove these results, we proposed two new outer-bounds, one for the single direct link feedback model and another for the feedback model with all four feedback links. The two new outer bounds together with the cut-set bound form a comprehensive outer bound for all feedback models, which allow for exact/approximate capacity region calculations for deterministic/Gaussian channel models.

In the weak interference regime, two new achievable strategies are proposed: one which is based on Han-Kobayashi type message splitting and the other which is based on block-Markov coding (at one transmitter) and dirty paper coding (at the other transmitter). Together, the two strategies achieve the exact/approximate capacity region for all 9 canonical feedback models for deterministic/Gaussian channels. In the achievable strategy involving Han-Kobayashi type message splitting, the transmitted message from each of the transmitters is split into two parts: private and common. The common part of the message of one of the transmitters is transmitted twice: once by the transmitter, which generates it, and once again (in the subsequent block) by the other transmitter after decoding it. The rate of the common message, which is re-transmitted is finely tuned so that it is decodable at the intended receiver after its first transmission, while it is decodable at the interfering receiver only after its second transmission. Although the common message first causes interference at one of the receivers, it allows for higher communication rates after interference resolution in the subsequent block. In the achievable strategy involving block-Markov encoding and dirty paper encoding/decoding, one of the transmitters employs block-Markov encoding, thereby correlating the interference it generates over blocks. The other transmitter knows the channel output via feedback, and using the knowledge of correlation of interference, it encodes its message using dirty paper coding to make its intended signal robust against future interference.

In the strong interference regime, feedback helps create a relay route, which is better than the direct channel from a transmitter to its intended receiver. The messages generated at a transmitter are first passed on to the interfering receiver. The interfering receiver then passes it on to its own transmitter (via feedback), which can then relay it

to the intended receiver. This way the intended receiver receives the message through an alternate path. Since the interference is stronger than the direct channel, relaying of messages can support higher rates than otherwise.

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Appendix

A. *Proof of Lemma 4.5*

Let $V_{1i} = \mathbf{S}^{q-m} X_{1i}$ and $V_{2i} = \mathbf{S}^{q-m} X_{2i}$. We know that X_{1i} and X_{2i} are given by

$$X_{1i} = f_{1i}(W_1, Y_1^{i-1}),\ X_{2i} = f_{2i}(W_2)$$

where $f_{1i}(.), f_{2i}(.)$ are some deterministic functions. We have

$$\begin{aligned}
& N(2R_1 + R_2) \\
& \quad \leq \quad 2H(W_1) + H(W_2) \\
& \quad \overset{\text{(a)}}{=} \quad H(W_1) + H(W_1|W_2) + H(W_2) \\
& \quad \overset{\text{(Fano)}}{\leq} \quad I(W_1; Y_1^N) + I(W_1; Y_1^N|W_2) + I(W_2; Y_2^N) + N(\epsilon_{1N} + \epsilon_{2N} + \epsilon_{3N}) \\
& \quad = \quad H(Y_1^N) - H(Y_1^N|W_1) + H(Y_1^N|W_2) - H(Y_1^N|W_1 W_2) + \\
& \qquad\quad H(Y_2^N) - H(Y_2^N|W_2) + N\epsilon_N,
\end{aligned} \tag{69}$$

where ϵ_{1N}, ϵ_{2N} and ϵ_{3N} correspond to the Fano's inequality applied to three different entropy terms, and $\epsilon_N = 3\max(\epsilon_{1N}, \epsilon_{2N}, \epsilon_{3N})$ and (a) holds because W_1 and W_2 are independent. Rearranging (69) yields

$$\begin{aligned}
&N(2R_1+R_2)\\
&\leq\quad H(Y_1^N)+\underbrace{H(Y_2^N)-H(Y_1^N|W_1)}+\underbrace{H(Y_1^N|W_2)-H(Y_1^N|W_1W_2)-H(Y_2^N|W_2)}+N\epsilon_N\\
&\stackrel{(b)}{\leq}\quad H(Y_1^N)+\underbrace{H(Y_2^N)+H(V_2^N|Y_2^N)-H(Y_1^N|W_1)}+\underbrace{H(Y_1^N|W_2)-H(Y_2^N|W_2)}+N\epsilon_N,
\end{aligned}$$

where (b) is true as entropy for discrete random variables is always positive. The three sub-expressions are independently bounded. The first sub-expression satisfies $H(Y_1^N) = \sum_{i=1}^N H(Y_{1i}|Y_1^{i-1}) \leq \sum_{i=1}^N H(Y_{1i})$ due to the chain rule of entropy followed by the fact that removing conditioning does not reduce entropy. The second sub-expression is bounded as follows:

$$H(Y_2^N)+H(V_2^N|Y_2^N)-H(Y_1^N|W_1)=H(Y_2^N|V_2^N)+H(V_2^N)-H(Y_1^N|W_1) \tag{70}$$

Observe the following:

$$\begin{aligned}
&H(V_2^N)-H(Y_1^N|W_1)\\
&\stackrel{(c)}{=}\quad H(V_2^N|W_1)-\sum_{i=1}^N H(Y_{1i}|W_1,Y_1^{i-1})\\
&\stackrel{(d)}{=}\quad H(V_2^N|W_1)-\sum_{i=1}^N H(Y_{1i}|W_1,Y_1^{i-1},X_1^i)\\
&\stackrel{(e)}{=}\quad H(V_2^N|W_1)-\sum_{i=1}^N H(V_{2i}|W_1,V_2^{i-1},Y_1^{i-1},X_1^i)\\
&\stackrel{(f)}{=}\quad \sum_{i=1}^N H(V_{2i}|W_1,V_2^{i-1})-\sum_{i=1}^N H(V_{2i}|W_1,V_2^{i-1},X_1^i,Y_1^{i-1})\\
&=\quad \sum_{i=1}^N I(V_{2i};X_1^i,Y_1^{i-1}|W_1,V_2^{i-1})\\
&=\quad \sum_{i=1}^N [H(X_1^i|W_1,V_2^{i-1})+H(Y_1^{i-1}|W_1,V_2^{i-1},X_1^i)]-[H(X_1^i|W_1,V_2^i)+H(Y_1^{i-1}|W_1,V_2^i,X_1^i)]\\
&\stackrel{(g)}{=}\quad \sum_{i=1}^N [H(X_1^i|W_1,V_2^{i-1})-H(X_1^i|W_1,V_2^i)]\\
&=\quad \sum_{i=1}^N I(X_1^i;V_{2i}|W_1,V_2^{i-1})\\
&\stackrel{(h)}{=}\quad \sum_{i=1}^N I(f(V_2^{i-1},W_1);V_{2i}|W_1,V_2^{i-1})\\
&=\quad 0,
\end{aligned} \tag{71}$$

where (c) is true because V_2^N depends only on W_2 and thus independent of W_1, (d) holds because X_1^i is a deterministic function of W_1 and Y_1^{i-1}, (e) is justified because $Y_{1i} = X_{1i} + V_{2i}$, (f) is due to the chain rule of entropy, (g) holds because Y_1^{i-1} is a deterministic function of X_1^{i-1} and V_2^{i-1}, (h) is true because of the chain rule because of the following: X_{1i} depends on W_1 and Y_1^{i-1}, but $Y_{1i} = X_{1i-1} + V_{2i-1}$. Thus X_{1i} is function of W_1, V_{2i-1}, and Y_1^{i-2}. This implies that X_1^i is a function of W_1 and V_2^{i-1} only. Combining (70) and (71), we have

$$H(Y_2^N)+H(V_2^N|Y_2^N)-H(Y_1^N|W_1)=H(Y_2^N|V_2^N)=\sum_{i=1}^N H(Y_{2i}|V_{2i},Y_2^{i-1},V_2^{i-1})\leq\sum_{i=1}^N H(Y_{2i}|V_{2i}), \tag{72}$$

where the inequality follows from the fact that removing conditioning cannot decrease entropy.

TABLE IX
ENCODING OF MESSAGES IN THE WEAK INTERFERENCE REGIME FOR THE (1000) FEEDBACK MODEL

	Block 1	Block i	Block B
Message X_{1i} at T_1	$\mathbf{0}_n^T$	$[X_{1i,c}^T, X_{2i-1,c}^T, X_{1i,p}^T]^T$	$[X_{1B,c}^T, X_{2B-1,c}^T, X_{1B,p}^T]^T$
Message X_{2i} at T_2	$[X_{21,c}^T, \mathbf{0}_l^T, X_{21,p}^T]^T$	$[X_{2i,c}^T, \mathbf{0}_l^T, X_{2i,p}^T]^T$	$\mathbf{0}_n^T$

Finally, for the third sub-expression, we have

$$\begin{aligned}
&H(Y_1^N|W_2) - H(Y_2^N|W_2)\\
&= H(Y_1^N|W_2) + H(Y_1^N|Y_2^N, W_2) - H(Y_1^N, Y_2^N|W_2)\\
&= H(Y_1^N|Y_2^N, W_2) - H(Y_2^N|Y_1^N, W_2)\\
&\leq H(Y_1^N|Y_2^N, W_2)\\
&\overset{\text{(j)}}{=} \sum_{i=1}^{N} H(Y_{1i}|Y_2^N, Y_1^{i-1}, W_2)\\
&\overset{\text{(k)}}{=} \sum_{i=1}^{N} H(Y_{1i}|Y_2^N, Y_1^{i-1}, W_2, X_2^i, V_2^i, V_1^i)\\
&\overset{\text{(l)}}{\leq} \sum_{i=1}^{N} H(Y_{1i}|V_{1i}, V_{2i})
\end{aligned} \tag{73}$$

(j) follows from the chain rule of entropy, (k) follows from the observation that X_2^i is a function of only (W_2, Y_1^{i-1}, Y_2^i), V_2^i is function of X_2^i, and V_1^i is a function of (X_2^i, Y_2^i), and (l) follows since conditioning reduces entropy.

Now combining all the expressions together we have

$$N(2R_1 + R_2) \leq \sum_{i=1}^{N}(H(Y_{1i}) + H(Y_{2i}|V_{2i}) + H(Y_{1i}|V_{1i}, V_{2i}) + N\epsilon_N \tag{74}$$

By randomization of time indices and letting $\epsilon_N \to 0$ as $N \to \infty$, we get

$$2R_1 + R_2 \leq H(Y_1) + H(Y_2|V_2) + H(Y_1|V_1, V_2). \tag{75}$$

The RHS is maximized when X_1 and X_2 are drawn from an i.i.d. distribution over $\mathbb{F}_2^q$, where each entry of the q-bit vector is i.i.d. $\mathrm{Bern}(\frac{1}{2})$. This gives us the outer bound as in the statement of Lemma 4.5.

B. *Achievable strategy for the corner points of the capacity region of the* (1000) *feedback model*

Encoding: At the u^{th} transmitter T_u, in the i^{th} block two i.i.d. bit vectors $X_{ui,c}$ and $X_{ui,p}$ are generated. The total number of transmission blocks is B. Let $\mathbf{0}_l = [0, 0, \dots 0]$ such that $|\mathbf{0}_l| = l$. The encoding of messages is for all the B blocks is shown in Table IX.

1) Weak interference regime $n \geq m$: In the weak interference regime, we note that $|X_{1i}| = |X_{2i}| = n$. The encoding scheme is complete, if the cardinality of $X_{1i,c}, X_{2i,c}, X_{1i,p}$ and $X_{2i,p}$ are specified.

Decoding: To allow reliable decoding, we specify the cardinality of the common and private message for corner points $\mathcal{K}_{\text{A}}$, $\mathcal{K}_{\text{B}}$, $\mathcal{K}_{\text{C}}$, $\mathcal{K}_{\text{D}}$ (19) as the following:

a) Corner point $\mathcal{K}_{\text{A}}$: When $m < \frac{n}{2}$, the desired corner point is $(n, n-2m)$, and it is achievable without feedback [22]. When $\frac{n}{2} \leq m \leq n$, the desired corner point is $(n, 0)$, which is trivially achievable without feedback.

b) Corner point $\mathcal{K}_{\text{C}}$: When $m < \frac{n}{2}$, the intersection is at the corner point $(n-m, n)$, which is identical to the corner point $\mathcal{K}_{\text{D}}$ that will be shown to achievable in Appendix B1c. When $\frac{2n}{3} \leq m \leq n$, we conclude from Lemma 4.4 and Theorem 2.1 that the corner point is achievable without feedback. When $\frac{n}{2} \leq m < \frac{2n}{3}$, the corner point $(m, 2n-2m)$ can be achieved as $B \to \infty$, if

$$|X_{1i,c}| = 2m - n,\ |X_{2i,c}| = n - m,\ |X_{1i,p}| = |X_{2i,p}| = n - m,\ \mathbf{0}_l = l = 2m - n. \tag{76}$$

c) Corner point $\mathcal{K}_{\mathrm{D}}$: The corner point $(n-m,n)$ can be achieved as $B \to \infty$ if

$$|X_{1i,c}|=0,\ |X_{2i,c}|=m,\ |X_{1i,p}|=|X_{2i,p}|=n-m. \tag{77}$$

D_1 and D_2 respectively perform backward and forward decoding. Due to backward decoding at D_1, before decoding the i^{th} block $X_{2i,c}$ is known. Thus, $X_{2i,c}$ can be subtracted from Y_{1i}, after which $X_{1i,c}, X_{2i-1,c}$ and $X_{1i,p}$ can be decoded. Due to forward decoding at D_2, while decoding the i^{th} block $X_{2i-1,c}$ is already known. Thus, $X_{2i-1,c}$ can be subtracted from Y_{2i}, after which $X_{2i,c}, X_{1i,c}$ and $X_{2i,p}$ can be decoded.

2) Strong interference regime $n<m$: In the strong interference regime, $|X_{1i}|=|X_{2i}|=m$. Proving achievability for the following two corner points is sufficient to show the achievability of the outer-bound.

a) Corner point $\mathcal{K}_{\mathrm{B}}$: The desired corner point is $(n, m-n)$, which is achievable without feedback for $n<m\le 2n$ [22]. For $m>2n$, the rate pair $(n,m-n)$ can be achieved if

$$|X_{1i,c}|=n\ |X_{2i,r}|=m-n,\ |X_{1i,p}|=|X_{2i,p}|=0,\ \mathbf{0}_l=l=n. \tag{78}$$

b) Corner point $\mathcal{K}_{\mathrm{D}}$: In this case, the desired corner point is $(0,m)$. It can be achieved if

$$|X_{1i,c}|=0,\ |X_{2i,c}|=m,\ |X_{1i,p}|=|X_{2i,p}|=0. \tag{79}$$

In this case, the unit D_1-feedback-T_1 entirely serves as a relay node. Forward decoding at both receivers is used to decode the desired messages.

C. *Proof of Theorem 5.2*

Let's define $S_{1i}=g_cX_{1i}+Z_{2i}$ and $S_{2i}=g_cX_{2i}+Z_{1i}$

$$\begin{aligned}
&N(R_1+R_2)\\
&\quad\le\quad H(W_1,W_2)=H(W_1|W_2)+H(W_2) &(80)\\
&\quad\overset{\text{(Fano)}}{\le}\quad I(W_1;Y_1^N|W_2)+I(W_2;Y_2^N)+N(\epsilon_{1N}+\epsilon_{2N})\\
&\quad=\quad \underbrace{h(Y_1^N|W_2)-h(Y_1^N|W_1,W_2)-h(Y_2^N|W_2)}+h(Y_2^N)+N\epsilon_N &(81)
\end{aligned}$$

where $\epsilon_{1N},\epsilon_{2N}$ appear after applying Fano's inequality to the two entropy terms in (80). Also, $\epsilon_N=\epsilon_{1N}+\epsilon_{2N}$. We now bound the expression in the under-brace in (81) as

$$\begin{aligned}
&h(Y_1^N|W_2)-h(Y_1^N|W_1,W_2)-h(Y_2^N|W_2)\\
&\overset{\text{(a)}}{\le}\quad h(Y_1^N|W_2)+h(Y_1^N|Y_2^N,W_2)-h(Y_1^N|Y_2^N,W_2)-h(Y_2^N|W_2)-\sum_{j=1}^{N}h(Z_{1i})\\
&=\quad h(Y_1^N|W_2)+h(Y_1^N|Y_2^N,W_2)-h(Y_1^N,Y_2^N|W_2)-\sum_{i=1}^{N}h(Z_{1i})\\
&=\quad h(Y_1^N|Y_2^N,W_2)-h(Y_2^N|Y_1^N,W_2)-\sum_{i=1}^{N}h(Z_{1i})\\
&\overset{\text{(b)}}{\le}\quad h(Y_1^N|Y_2^N,W_2)-\sum_{i=1}^{N}[h(Z_{1i})+h(Z_{2i})]\\
&\overset{\text{(c)}}{=}\quad \sum_{i=1}^{N}h(Y_{1i}|Y_2^N,W_2,Y_1^{i-1})-\sum_{i=1}^{N}[h(Z_{1i})+h(Z_{2i})]
\end{aligned}$$

$$\begin{aligned}
&\overset{\text{(d)}}{=}\quad \sum_{i=1}^{N}h(Y_{1i}|Y_2^N,W_2,Y_1^{i-1},X_2^i,S_{1i})-\sum_{i=1}^{N}[h(Z_{1i})+h(Z_{2i})]\\
&\overset{\text{(e)}}{\le}\quad \sum_{i=1}^{N}h(Y_{1i}|X_{2i},S_{1i})-\sum_{i=1}^{N}[h(Z_{1i})+h(Z_{2i})] &(82)
\end{aligned}$$

where (a) holds since

$$\begin{aligned} h(Y_1^N|W_1,W_2) &= \sum_{i=1}^N h(Y_{1i}|W_1,W_2,Y_1^{i-1}) \geq \sum_{i=1}^N h(Y_{1i}|W_1,W_2,Y_1^{i-1},Y_2^{i-1}) \\ &= \sum_{i=1}^N h(Y_{1i}|W_1,W_2,Y_1^{i-1},Y_2^{i-1},X_{1i},X_{2i}) = \sum_{i=1}^N h(Z_{1i}), \end{aligned}$$

(b) follows from

$$\begin{aligned} h(Y_2^N|Y_1^N,W_2) &= \sum_{i=1}^N h(Y_{2i}|Y_1^N,W_2,Y_2^{i-1}) = \sum_{i=1}^N h(Y_{2i}|Y_1^N,W_2,Y_2^{i-1},X_{2i}) \\ &\geq \sum_{i=1}^N h(Y_{2i}|Y_1^N,W_2,Y_2^{i-1},X_{2i},X_{1i}) = \sum_{i=1}^N h(Z_{2i}), \end{aligned}$$

(c) is due to the chain rule of entropy, (d) holds because given W_2 and X_2^N can be precisely determined and $Y_{2i} = X_{2i} + S_{1i}$ and thus given Y_{2i} and X_{2i}, S_{2i} can be precisely determined, (e) uses the fact that removing conditioning does not increase the entropy.

We plug-in this part in the original sum-rate bound (81) to get

$$R_1 + R_2 \leq \frac{1}{N}\left(h(Y_2^N) + \sum_{i=1}^N h(Y_{1i}|X_{2i},S_{1i}) - \sum_{i=1}^N [h(Z_{1i} + h(Z_{2i})]\right) + \epsilon_N$$

Letting $N \to \infty$ we can make $\epsilon_N \to 0$. Moreover applying the chain rule of entropy and noting that removing conditioning does not increase entropy, the following outer bound is obtained

$$R_1 + R_2 \leq \frac{1}{N}\left(\sum_{i=1}^N h(Y_{2i}) + \sum_{i=1}^N h(Y_{1i}|X_{2i},S_{1i}) - \sum_{i=1}^N [h(Z_{1i}) + h(Z_{2i})]\right)$$

By simply interchanging the indices of the users, i.e., following the substitution $1 \to 2$ and vice versa, we obtain

$$R_1 + R_2 \leq \frac{1}{N}\left(\sum_{i=1}^N h(Y_{1i}) + \sum_{i=1}^N h(Y_{2i}|X_{1i},S_{2i}) - \sum_{i=1}^N [h(Z_{1i}) + h(Z_{2i})]\right) \tag{83}$$

Assuming that both X_1 and X_2 is drawn from complex Gaussian distributions with mean 0 and variance 1, and the correlation between X_1 and X_2 is ρ, i.e. $\rho = \mathsf{E}[X_1 X_2{}^*]$, and then the (83) can be expressed in terms of SNR and INR as

$$R_1 + R_2 \leq \sup_{0\leq|\rho|\leq 1}\left\{\log\left(1 + \frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right) + \log(1 + \mathsf{SNR} + \mathsf{INR} + 2|\rho|\sqrt{\mathsf{SNR.INR}})\right\}. \tag{84}$$

which is the statement of Theorem 5.2.

D. *Proof of Theorem 5.3*

$$\begin{aligned} & N(2R_1 + R_2) \\ & \overset{\text{(a)}}{=} H(W_1) + H(W_1|W_2) + H(W_2) \\ & \overset{\text{(b)}}{\leq} I(W_1;Y_1^N) + I(W_1;Y_1^N|W_2) + I(W_2;Y_2^N) + N(\epsilon_{1N} + \epsilon_{2N} + \epsilon_{3N}) \\ & = h(Y_1^N) - h(Y_1^N|W_1) + h(Y_1^N|W_2) - h(Y_1^N|W_1W_2) + h(Y_2^N) - h(Y_2^N|W_2) + N\epsilon_N \end{aligned} \tag{85}$$

(a) is due to the independence of the messages at the two transmitters. (b) follows due to applying Fano's inequality to each of the entropy terms and $\epsilon_N = 3\max(\epsilon_{1N}, \epsilon_{2N}, \epsilon_{3N})$. Rearranging the terms, the following expression is obtained

$$h(Y_1^N) + \underbrace{h(Y_2^N) - h(Y_1^N|W_1)} + \underbrace{h(Y_1^N|W_2) - h(Y_1^N|W_1W_2) - h(Y_2^N|W_2)} + N\epsilon_N \tag{86}$$

The three sub-expressions are separately bounded. The first sub-expression is $h(Y_1^N) = \sum_{i=1}^N h(Y_{1i}|Y_1^{i-1}) \leq \sum_{i=1}^N h(Y_{1i})$, because removing conditioning does not reduce entropy.

In order to bound the second sub-expression, observe the following:

$$\begin{aligned}
& h(S_2^N) - h(Y_1^N|W_1) \\
&= h(S_2^N) - \sum_{i=1}^N h(Y_{1i}|W_1 Y_1^{i-1}) \\
&\overset{(c)}{=} h(S_2^N) - \sum_{i=1}^N h(Y_{1i}|W_1 Y_1^{i-1} X_1^i) \\
&\overset{(d)}{=} h(S_2^N|W_1) - \sum_{i=1}^N h(S_{2i}|W_1 Y_1^{i-1} X_1^i) \\
&\overset{(e)}{=} \sum_{i=1}^N h(S_{2i}|W_1 S_2^{i-1}) - \sum_{i=1}^N h(S_{2i}|W_1 Y_1^{i-1} X_1^i) \\
&\overset{(f)}{=} \sum_{i=1}^N h(S_{2i}|W_1 S_2^{i-1}) - \sum_{i=1}^N h(S_{2i}|W_1 Y_1^{i-1} X_1^i S_2^{i-1}) \\
&= \sum_{i=1}^N I(S_{2i}; Y_1^{i-1} X_1^i | S_2^{i-1} W_1) \\
&= \sum_{i=1}^N h(X_1^i Y_1^{i-1}|S_2^{i-1} W_1) - h(X_1^i Y_1^{i-1}|S_2^i W_1) \\
&= \sum_{i=1}^N h(X_1^i|S_2^{i-1} W_1) + h(Y_1^{i-1}|X_1^i S_2^{i-1} W_1) - (h(X_1^i|S_2^i W_1) + h(Y_1^{i-1}|X_1^i S_2^i W_1)) \\
&\overset{(g)}{=} \sum_{i=1}^N h(X_1^i|S_2^{i-1} W_1) - \sum_{i=1}^N h(X_1^i|S_2^i W_1) \\
&\overset{(h)}{=} \sum_{i=1}^N \sum_{j=1}^i h(X_{1j}|X_1^{j-1}, S_2^{i-1}, W_1) - \sum_{i=1}^N \sum_{j=1}^i h(X_{1j}|X_1^{j-1}, S_2^i, W_1) \\
&\overset{(i)}{=} \sum_{i=1}^N \sum_{j=1}^i h(X_{1j}|X_1^{j-1}, S_2^{i-1}, W_1, Y_1^{j-1}) - \sum_{i=1}^N \sum_{j=1}^i h(X_{1j}|X_1^{j-1}, S_2^i, W_1, Y_1^{j-1}) \\
&\overset{(j)}{=} 0
\end{aligned} \tag{87}$$

(c) holds because X_1^i is a deterministic function of W_1 and Y_1^{i-1}, (d) is justified because the message W_1 is independent of W_2, and S_{2i} depends only on W_2 and the noise Z_{1i}, (e) holds due to the chain rule of entropy (f) is because S_2^{i-1} can be precisely determined from X_1^i and Y_1^{i-1} (g) holds because given X_1^{i-1} and S_2^{i-1}, Y_1^{i-1} can be precisely determined, (h) is obtained by applying the chain rule of entropy to both of the summation terms, (i) holds as Y_1^{j-1} can be precisely determined using X_1^{j-1} and S_2^{j-1}, (j) is true because given W_1 and Y_1^{j-1}, X_{1j} can be precisely determined and hence the value of each of the entropy terms is 0.

Let us introduce $S'_{2i} = g_c X_{2i} + Z'_{2i}$, where for every i, the Z'_{2i} is independently distributed with $\mathcal{CN}(0,1)$. Since entropy is a function of the probability density function, $h(S_2^N) = h(S_2'^N)$. From (87), we know that $h(Y_1^N|W_1) = h(S_2^N)$. Thus, $h(Y_1^N|W_1) = h(S_2^N) = h(S_2'^N)$, which can be used in the second subexpression in

(86) such that

$$\begin{aligned} & h(Y_2^N) - h(Y_1^N|W_1) \\ & = h(Y_2^N) - h(S_2'^N) \\ & = h(Y_2^N) + h(S_2'^N|Y_2^N) - h(S_2'^N|Y_2^N) - h(S_2'^N) \\ & = h(Y_2^N, S_2'^N) - h(S_2'^N) - h(S_2'^N|Y_2^N) \\ & \overset{(a)}{\leq} h(Y_2^N|S_2'^N) - h(S_2'^N|Y_2^N, X_2^N) \\ & \overset{(b)}{=} h(Y_2^N|S_2'^N) - h(Z_2'^N|Y_2^N, X_2^N) \\ & \overset{(c)}{=} h(Y_2^N|S_2'^N) - h(Z_2'^N) \\ & \overset{(d)}{\leq} \sum_{i=1}^{N}(h(Y_{2i}|S'_{2i}) - h(Z'_{2i})) \end{aligned}$$

where (a) holds because conditioning reduces entropy, (b) holds because $S_2'^N$ is a function of X_2^N and $Z_2'^N$, (c) holds because $Z_2'^N$ is independent of (Y_2^N, X_2^N), (d) holds because entropy can only increase if conditioning is removed and noise Z'_{2i} is independent of Z'_{2j} for $i \neq j$.

The third subexpression in (86) is also bounded with (82). Putting them together, we finally have the following bound

$$N(2R_1 + R_2) \leq \sum_{i=1}^{N}(h(Y_{1i}) + h(Y_{1i}|S_{1i}X_{2i}) - h(Z_{1i}) - h(Z_{2i}) + h(Y_{2i}|S'_{2i}) - h(Z'_{2i})) + N\epsilon_N.$$

Again letting $N \to \infty$ we can make $\epsilon_N \to 0$ and thus we have the upper bound

$$2R_1 + R_2 \leq \frac{1}{N}\left(\sum_{i=1}^{N}[h(Y_{1i}) + h(Y_{1i}|S_{1i}X_{2i}) + h(Y_{2i}|S'_{2i}) - h(Z_{1i}) - h(Z_{2i}) - h(Z'_{2i})]\right). \tag{88}$$

Assuming that both X_1 and X_2 is drawn from complex Gaussian distributions with mean 0 and variance 1, and the correlation between X_1 and X_2 is ρ, i.e. $\rho = \mathsf{E}[X_1X_2{}^*]$, and then the (88) can be expressed in terms of SNR and INR as

$$\begin{aligned} 2R_1 + R_2 \leq & \sup_{0\leq|\rho|\leq 1}\{\log\left(1 + \frac{(1-|\rho|^2)\mathsf{SNR}}{1+(1-|\rho|^2)\mathsf{INR}}\right) + \log(1 + \mathsf{SNR} + \mathsf{INR} + 2|\rho|\sqrt{\mathsf{SNR.INR}}) \\ & + \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - (1+|\rho|^2)\mathsf{INR} + 2|\rho|\sqrt{\mathsf{SNR.INR}}}{1+\mathsf{INR}}\right)\} \end{aligned} \tag{89}$$

which is the statement of the Theorem 5.3.

E. Gap to Capacity

Corresponding to the relevant corner point, we show the gap of the achievable rate pairs described in Table VI. First, we bound the gap for $\alpha \in [0, 1]$ and then for $\alpha \in (1, \infty)$.

1) Corner point $\overline{\mathcal{K}}_{\mathrm{A}}$ *for* (1000) *feedback model:* It is sufficient to consider only two interference regimes. The achievable rate pair is described by the corner point $\mathcal{P}_{\mathrm{A}}$ in Table VI, which is achievable without feedback.

a) $\alpha \in \left[0, \frac{1}{2}\right)$*:* The gaps of the achievable rate R_2 from the outer bound is

$$\begin{aligned} \overline{C}_2 - R_2 \leq & \log\left(1 + \frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1 + \mathsf{SNR} + \mathsf{INR} + 2\sqrt{\mathsf{SNR.INR}}) \\ & + \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{1+\mathsf{INR}}\right) - 2\log(1+\mathsf{SNR}) - \log\left(\frac{\mathsf{SNR}}{2\mathsf{INR}^2}\right) \leq 3 + \log(3) \end{aligned} \tag{90}$$

and

$$\overline{C}_1 - R_1 \leq \log(1+\mathsf{SNR}) - [\log(\mathsf{SNR}) - 1] \leq \log\left(\frac{1+\mathsf{SNR}}{\mathsf{SNR}}\right) + 1 = 2. \tag{91}$$

b) $\alpha \in \left[\frac{1}{2}, 1\right]$*:* The achievable rates and the corresponding distances from the outer bounds are

$$\begin{aligned}\overline{C}_2 - R_2 \quad &\leq \quad \log\left(1 + \frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1 + \mathsf{SNR} + \mathsf{INR} + 2\sqrt{\mathsf{SNR.INR}}) \\ &\quad + \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{1 + \mathsf{INR}}\right) - 2\log(1 + \mathsf{SNR}) \leq 3. \end{aligned} \tag{92}$$

Both the outer and inner bounds for R_1 are $\log(1 + \mathsf{SNR})$ and thus the gap is 0. The point $(\log(1 + \mathsf{SNR}), 0)$ is trivially achievable.

2) Corner point $\overline{\mathcal{K}}_{\mathrm{C}}$ *for* (1000) *feedback model:* The gap between the corner point $\mathcal{P}_{\mathrm{C}}$ in Table VI and the outer bound $\overline{\mathcal{K}}_{\mathrm{C}}$, is computed separately for three different regimes of interference.

a) $\alpha \in \left[0, \frac{1}{2}\right)$*:* The distance of the outer bound from the achievable rate R_2 is

$$\begin{aligned}\overline{C}_2 - R_2 \quad &\leq \quad \left[\log\left(1 + \frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log\left(1 + \mathsf{SNR} + \mathsf{INR} + 2\sqrt{\mathsf{SNR.INR}}\right) - \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{1 + \mathsf{INR}}\right)\right] \\ &\quad - \left[\log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) + \log(\mathsf{INR}) - \log(3)\right] \leq 1 + 2\log(3). \end{aligned} \tag{93}$$

The corresponding gap for the achievable rate R_1 is

$$\overline{C}_1 - R_1 \quad \leq \quad \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{\mathsf{INR} + 1}\right) - \log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) \leq 2. \tag{94}$$

b) $\alpha \in \left[\frac{1}{2}, \frac{2}{3}\right)$*:* The distance of R_2 from the outer bound is bounded as follows

$$\begin{aligned}\overline{C}_2 - R_2 \quad &\leq \quad \log\left(1 + \frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1 + \mathsf{SNR} + \mathsf{INR} + 2\sqrt{\mathsf{SNR.INR}}) - \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{\mathsf{INR} + 1}\right) \\ &\quad - \left[\log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) + \log\left(1 + \frac{\mathsf{SNR}}{\mathsf{INR}}\right) - \log(4)\right] \leq 3 + \log(3) \end{aligned} \tag{95}$$

and the distance of R_1 from the outer bound is bounded as

$$\begin{aligned}\overline{C}_1 - R_1 \quad &= \quad \left[\log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{\mathsf{INR} + 1}\right)\right] - \left[\log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) + \log\left(1 + \frac{\mathsf{INR}^2}{\mathsf{SNR}}\right) - \log(4)\right] \\ &\leq \quad 3. \end{aligned} \tag{96}$$

c) $\alpha \in \left[\frac{2}{3}, 1\right]$*:* The distance of R_2 from its corresponding outer bound is

$$\begin{aligned}\overline{C}_2 - R_2 \quad &\leq \quad \log\left(1 + \frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1 + \mathsf{SNR} + \mathsf{INR} + 2\sqrt{\mathsf{SNR.INR}}) - \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{\mathsf{INR} + 1}\right) \\ &\quad - \left[\log\left(\frac{\mathsf{SNR}}{\mathsf{INR}}\right) + \log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) - \log(1.5)\right] \leq 2 + \log(3). \end{aligned} \tag{97}$$

and the corresponding gap between the outer bound and R_1 is given by

$$\begin{aligned}\overline{C}_1 - R_1 \quad &\leq \quad \log\left(1 + \mathsf{INR} + \frac{\mathsf{SNR} - \mathsf{INR}}{\mathsf{INR} + 1}\right) - [\log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) + \log\left(\frac{\mathsf{INR}^2}{\mathsf{SNR}}\right) - \log(3)] \\ &\leq \quad 2 + \log(3). \end{aligned} \tag{98}$$

3) Corner point $\overline{\mathcal{K}}_{\mathrm{D}}$ *for* (1111) *feedback model:* Note that the corner point $\overline{\mathcal{K}}_{\mathrm{D}}$ is identical for all (1xxx) feedback models. This corner point is within a constant gap from the achievable rate pair described by the corner point $\mathcal{P}_{\mathrm{D}}$ in Table VI.

a) $\alpha \in [0, 1]$*:* The gap between the achievable rate R_2 from $\overline{C}_2$ is

$$\overline{C}_2 - R_2 \leq \log(1 + \mathsf{SNR} + \mathsf{INR}) - \log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) - \log(\mathsf{INR}) + \log(3) \leq 1 + \log(3). \tag{99}$$

The corresponding gap of R_1 from $\overline{C}_1$ is

$$\overline{C_1} - R_1 \quad \leq \quad \left[\log\left(1 + \frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log\left(\frac{1 + \mathsf{SNR} + \mathsf{INR} + 2\sqrt{\mathsf{SNR.INR}}}{1 + \mathsf{SNR} + \mathsf{INR}}\right)\right] - \log\left(1 + \frac{\mathsf{SNR}}{2\mathsf{INR}}\right) \leq 2. \tag{100}$$

4) Corner point $\overline{\mathcal{K}}_{\mathrm{D}}$ of (0110) feedback model: Note that the corner point $\overline{\mathcal{K}}_{\mathrm{D}}$ is identical for (0110) and (0010) feedback models. The gap between the achievable rate R_2 from $\overline{C}_2$ is

$$\overline{C}_2 - R_2 \leq \log(1+\mathsf{SNR}) - \log\left(1+\frac{\mathsf{SNR}}{2\mathsf{INR}}\right) - \log(\mathsf{INR}) + \log(3) \leq 1 + \log(3). \tag{101}$$

The corresponding gap of R_1 from $\overline{C}_1$ is

$$\overline{C_1} - R_1 \quad \leq \quad \left[\log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log\left(\frac{1+\mathsf{SNR}+\mathsf{INR}+2\sqrt{\mathsf{SNR.INR}}}{1+\mathsf{SNR}+\mathsf{INR}}\right)\right] - \log\left(1+\frac{\mathsf{SNR}}{2\mathsf{INR}}\right) \leq 2. \tag{102}$$

5) Corner point $\overline{\mathcal{K}}_{\mathrm{B}}$ of (0110) feedback model: Note that the corner point $\overline{\mathcal{K}}_{\mathrm{B}}$ is identical for (0110) and (0010) feedback models. The gap of the achievable rate pair described by $\mathcal{P}_{\mathrm{B2}}$, in the weak interference regime, described in Table VIII is computed as follows

$$\begin{aligned}\overline{C}_2 - R_2 \quad \leq \quad & \log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1+\mathsf{SNR}+\mathsf{INR}+2\sqrt{\mathsf{SNR.INR}}) \\ & -\log(1+\mathsf{SNR}) - \min\{\log\left(\frac{\mathsf{SNR}}{2\mathsf{INR}}\right), \log\left(\frac{\mathsf{SNR}}{4\mathsf{INR}}\right)\} \leq 3 + \log(3)\end{aligned} \tag{103}$$

and the corresponding gap of R_1 from $\overline{C}_1$ is

$$\overline{C}_1 - R_1 \leq \log(1+\mathsf{SNR}) - \log\left(1+\frac{\mathsf{SNR}}{2}\right) \leq 1 \tag{104}$$

Now we list the gap for corner points where $\alpha \in (1,\infty)$.

6) Corner point $\overline{\mathcal{K}}_{\mathrm{B}}$ of (1000) feedback model: The achievability is described in Section V-B1, and the achievable rate pair is described by $\mathcal{P}_{\mathrm{B}}$ in Table VI.

a) $\alpha \in (1,2)$: The gap for the achievable rate R_2 is

$$\begin{aligned}\overline{C}_2 - R_2 \quad \leq \quad & \left[\log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1+\mathsf{SNR}+\mathsf{INR}+2\sqrt{\mathsf{SNR.INR}}) - \log(1+\mathsf{SNR})\right] - \log\left(1+\frac{\mathsf{INR}}{\mathsf{SNR}}\right) \\ \leq \quad & \log(2) + \log\left(\frac{\mathsf{SNR}+\mathsf{SNR}^2+\mathsf{SNR.INR}+2\sqrt{\mathsf{SNR.INR}}.\mathsf{SNR}}{\mathsf{SNR}^2+\mathsf{SNR}+\mathsf{SNR.INR}+\mathsf{INR}}\right) \leq 1+\log(3).\end{aligned} \tag{105}$$

The gap between the achievable rate R_1 and the outer bound is

$$\overline{C}_1 - R_1 \leq \log(1+\mathsf{SNR}) - \log(\mathsf{SNR}) \leq \log(2) = 1. \tag{106}$$

b) $\alpha \in [2,\infty)$: The outer bound on R_2 and its gap from the outer bound is

$$\begin{aligned}\overline{C}_2 - R_2 \quad \leq \quad & \left[\log(1+\mathsf{INR}+\mathsf{SNR}+2\sqrt{\mathsf{INR.SNR}}) + \log\left(1+\frac{\mathsf{SNR}}{\mathsf{INR}+1}\right) - \log(1+\mathsf{SNR})\right] - \left[\log\left(\frac{\mathsf{INR}}{\mathsf{SNR}}\right)\right] \\ \leq \quad & \log\left(\frac{\mathsf{SNR}+\mathsf{INR.SNR}+\mathsf{SNR}^2+2\sqrt{\mathsf{INR.SNR}}.\mathsf{SNR}}{\mathsf{INR}+\mathsf{INR.SNR}}\right) + 1 \leq \log(3)+1.\end{aligned} \tag{107}$$

The gap from the outer bound for the achievable rate R_1 is

$$\overline{C}_1 - R_1 \leq \log(1+\mathsf{SNR}) - [\log(\mathsf{SNR})] \leq \log\left(1+\frac{1}{\mathsf{SNR}}\right) \leq \log(2) = 1. \tag{108}$$

7) Corner point $\overline{\mathcal{K}}_{\mathrm{D}}$ of (1111) feedback model: The corner point $\overline{\mathcal{K}}_{\mathrm{D}}$ is identical for all (1xxx) feedback models. The achievable rate pair is described by $\mathcal{P}_{\mathrm{D}}$ in Table VI.

a) $\alpha \in (1,\infty)$: The gap between R_2 and the outer bound is

$$\overline{C}_2 - R_2 \leq \log(1+\mathsf{INR}+\mathsf{SNR}) - \log(1+\mathsf{INR}) \leq 1 \tag{109}$$

The achievable rate $R_1 = 0$ and the corresponding gap from the outer bound corner point is

$$\begin{aligned}\overline{C}_1 - R_1 &\leq \log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1+\mathsf{INR}+\mathsf{SNR}+2\sqrt{\mathsf{INR.SNR}}) - \log(1+\mathsf{INR}+\mathsf{SNR}) \\ &\leq \log(2) + \log\left(1+\frac{2\sqrt{\mathsf{SNR.INR}}}{1+\mathsf{INR}+\mathsf{SNR}}\right) \leq \log(2)+\log(2) = 2.\end{aligned} \tag{110}$$

8) Corner point $\overline{\mathcal{K}}_{\mathrm{D}}$ of (0110) feedback model: When $1 < \alpha < 2$, the rate pair described by $\mathcal{P}_{\mathrm{D}2}$ in Table VIII is achievable without any feedback. Its gap from $\overline{\mathcal{K}}_{\mathrm{D}}$ is computed as follows:

a) $\alpha \in (1,2)$:

$$\begin{aligned}\overline{C}_1 - R_1 &\leq \log\left(1+\frac{\mathsf{SNR}}{1+\mathsf{INR}}\right) + \log(1+\mathsf{SNR}+\mathsf{INR}+2\sqrt{\mathsf{SNR.INR}}) - \log(1+\mathsf{SNR}) - \log\left(1+\frac{\mathsf{INR}}{\mathsf{SNR}}\right) \\ &\leq \log(2)+\log(2)+\log\left(\frac{\mathsf{SNR}+\mathsf{SNR}^2+\mathsf{INR.SNR}+2\mathsf{SNR}.\sqrt{\mathsf{SNR.INR}}}{2\mathsf{SNR}+2\mathsf{SNR}^2+2\mathsf{INR}+2\mathsf{INR.SNR}}\right) \leq 2.\end{aligned} \tag{111}$$

$$\overline{C}_2 - R_2 \leq \log(1+\mathsf{SNR}) - \log(\mathsf{SNR}) \leq 1. \tag{112}$$